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FACTOR SUPPLY CHANGES IN SMALL OPEN ECONOMIES: RYBCZYNSKI DERIVATIVES UNDER INCREASING MARGINAL COSTS*

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We discuss the validity of Rybczynski's theorem under increasing marginal costs within firms or industries. In particular, we show that an extra supply of any factor may lead to an expansion of all sector outputs if at least one sector permits input substitution. We provide a corresponding necessary and sufficient condition. This condition can even be satisfied when the equilibrium is Walrasian and Marshallian stable. Our findings are also robust with respect to aggregate improvements in total factor productivity which raise the economy's outputs beyond private returns. (JEL: D50, F11)

1. Motivation

The famous theorem by Rybczynski (1955) is one of the single most significant insights of modern pure trade theory. Given the standard Heckscher-Ohlin framework of a small open economy with two commodities and factors, the theorem states that an extra supply of some input will induce the production sector where this input is used more intensively to expand more than proportionally. At the same time, the other sector will contract. Proofs of this theorem usually rely on the convenient simplification that scale effects can be assumed away due to constant returns in production. The underlying notion of a small economy is a relative concept. Under globalization, an economy or economic region may quickly become small relative to a growing world market.

Not surprising, the theorem does not in general carry over to variations of the base para-

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digm. Blackorby/Schworm/Venables (1993, p. 426) have shown that under a regime of joint production deviations from the usual Rybczynski adjustments may even occur for constant returns to scale. Other authors, starting from a seminal paper by Jones (1968), have introduced technologies with scale economies or diseconomies. An important feature of these technologies is that any improvement or deterioration in the overall productivity of inputs is considered as an output externality which is not within the exploitation potential of a single firm or industry. From the perspective of any one firm, though, the constant-returns hypothesis is maintained. The concept of perfectly competitive markets can thus be preserved. So-called comparative-statics paradoxes may then emerge and may sometimes persist despite supplementary conditions. The literature also indicates that these paradoxes do not occur, however, when the commodity markets are Marshallian stable (cf. Mayer (1974), Panagariya (1980), Ide/Takayama (1988, 1990), Ingene/Yu (1991), Ishikawa (1994), Burney (1996)).

The assumed constancy of internal returns to production can be motivated on a-priori grounds by a standard replication argument: A firm may always choose to reiterate its activities – such that, e.g., a duplication of the firm's inputs must result in twice as much output as before. Yet this popular view has been criticized by Buchanan/Yoon (1999, pp. 517) as converting a production function into an 'engineering tautology'. To quote Kemp (1969, p. 154):

> "The assumption of constant returns to scale [...] has been a very convenient assumption since it has enabled us to focus attention [...] It is conceivable, however, that Nature is blind to the comforts of academic economists."

In a more recent contribution, Choi (1999, p. 18), providing a number of further references, points out that the assumption of constant returns to scale

> "[...] precludes any divergence in industrial returns to scale, which have been recognized as an indisputable phenomenon since the early days of economic science."

Empirical support against the constant-returns assumption has been provided by the new literature on the total factor productivity of a large number of (mostly manufacturing) industries in Europe (Caballero/Lyons (1990, 1991), Oulton (1996), Midelfart/Steen (1999), Lindström (2000), Henriksen/Midelfart/Steen (2001)) as well as in Canada and in the United States (Caballero/Lyons (1992), Benarroch (1997)). While the main focus of this literature is on positive inter-industry externalities, many authors also found strong evidence of decreasing returns to scale within firms or industries or within regions (e.g., Caballero/Lyons (1990, 1991), Burnside (1996), Lindström (2000)).[1] We conclude that decreasing returns are a legitimate and even important topic to be analyzed in the context of the Rybczynski problem.

Therefore, our model of a small open and competitive economy with two or more goods and factors imposes the conjugate assumption that firms operate under *decreasing returns to scale*, i.e. with increasing marginal costs. This economy can be interpreted as a geographic region which is exposed to global competition. We assume that the firms in this region benefit from agglomeration advantages and knowledge spillovers which they perceive as an exogenous productivity gain. The model is otherwise fairly standard. In particular, following Woodland (1982), our analysis is carried out entirely in dual space. We start from (minimum) cost functions and, hence, exploit the structure and regularity imposed by duality theory irrespective of the functional form of the underlying technology.[2]

In contrast to the conclusions of earlier writers and Rybczynski's theorem notwithstanding, in our model *all* outputs may benefit simultaneously from larger endowments. As a notable exception in the literature, Ylönen (1987, pp. 214–216) already pointed out that such an outcome can arise – as a plausible implication –

[1] *The reader may consult Basu/Fernald (1995, 1997) for a critical literature assessment from a methodological perspective.*

[2] *Thereby, we need no longer refer to traditional, however less convenient, concepts such as the 'production possibility locus' of an economy (cf. Herberg/Kemp (1969)).*

from the Hansson/Lundahl (1983) model of a 2×2 economy with differentiable and homogeneous production functions, if the degrees of homogeneity are less than one for both goods. (Also cf. a respective brief remark by Hansson/Lundahl (1983, p. 538).) We provide *necessary and sufficient* conditions for our general result which requires that at least one production sector permits input substitution. Our conditions are illustrated by a selection of examples. These examples cover both substitution and non-substitution technologies, as well as technologies with a specific factor. We allow for inferior inputs like low-skilled labor, which do not exist under constant returns, and consider factor complements when more than two factors are present (cf. Diewert (1982, p. 569)). A further general result, which differs from both the 2×2 case with constant returns and the Hansson/Lundahl (1983) approach, states that *all* factor prices may fall when any single endowment is increased. Finally, and contrary to the literature, we show that our observations do *not* rule out the Walrasian and Marshallian stability of an equilibrium. Equally important, our findings are robust with respect to global shifts in total factor productivity which raise the economy's outputs beyond private returns.

The paper is organized as follows: Section 2 provides our major results on the existence of non-standard adjustments to endowment changes of a small competitive economy under increasing marginal costs. Two computational examples with different parameterizations are presented in Section 3 for the purpose of illustration. In Section 4, stability is shown. Section 5 introduces external scale effects, and Section 6 concludes.

2. A Small-Country Model with Increasing Marginal Costs

We consider $n \geq 2$ profit maximizing sectors or industries each producing $x_i > 0$ ($i = 1, \ldots, n$) units of a single output from at least one input. The economy's total number of inputs is $m \geq 2$. Every input shall be used for the fabrication of some good. There are fixed local factor endowments $\mathbf{v}' = (v_1, \ldots, v_m)$ where $v_j > 0$ for all $j = 1, \ldots, m$.[3] The small, open economy takes world output prices $p_i > 0$ ($i = 1, \ldots, n$) as exogenous whereas domestic factor prices $w_j > 0$ ($j = 1, \ldots, m$) are determined endogenously due to factor immobility. We also assume that the minimum cost in each sector i of producing x_i output units at factor prices $\mathbf{w}' = (w_1, \ldots, w_m)$ can be computed from a twice continuously differentiable cost function $C^i : \mathbb{R}^m_{++} \times \mathbb{R}_{++} \to \mathbb{R}_{++}$ with the following properties satisfied for all $(\mathbf{w}, x_i) \in \mathbb{R}^m_{++} \times \mathbb{R}_{++}$: Cost is a nondecreasing first-order homogeneous and (weakly) concave function of factor prices $\mathbf{w}$. Marginal cost is positive and (at least locally) increasing (because of decreasing internal returns to scale in production).[4]

The last-mentioned condition will be satisfied, for instance, if the marginal cost curve of a representative firm in a given industry and, hence, the firm's average cost curve are U-shaped. Then, the profit maximizing output of this firm is bounded from below by the positive output level which minimizes average cost at given factor prices. This output (the so-called efficient scale, Mas-Colell et al. (1995, p. 144)) will always be determined along the upward-bending part of the marginal cost curve and establishes a discrete market-entry barrier for new competitors.

Our analysis relies on the postulate that the economy attains an equilibrium state in which all factors are at full employment. Hence, $\Sigma_{i=1}^n C^i_{w_j}(\mathbf{w}, x_i) = v_j$ for all j, where $C^i_{w_j}$ is shorthand notation for $\partial C^i / \partial w_j$ and thus stands for the quantity demanded of factor j in sector i according to Shephard's Lemma. At the same time, marginal costs shall equal output prices in all sectors i: $C^i_{x_i}(\mathbf{w}, x_i) = p_i$. We assume that each industry supports a given number of firms which can operate at least at their efficient scale without loss. This number shall not be affected by small parameter changes. We are thus given $m + n$ model equations. Total differentiation yields:

[3] *By default, all vectors shall be defined as column vectors. The prime '′' denotes transposes.*

[4] *We note for a cost function which is continuously differentiable in factor prices that the isoquants of the underlying production function do not allow for linear substitution between inputs.*

$$(1)\quad \begin{pmatrix} \mathbf{C_{ww}} & \mathbf{C_{xw}} \\ \mathbf{C'_{xw}} & \mathbf{D} \end{pmatrix} \begin{pmatrix} d\mathbf{w} \\ d\mathbf{x} \end{pmatrix} = \begin{pmatrix} d\mathbf{v} \\ d\mathbf{p} \end{pmatrix},$$

introducing as **x** and **p** the vectors of sector outputs and output prices, respectively, and with matrices $\mathbf{C_{ww}} := \Sigma_{k=1}^{n} \mathbf{C}_{\mathbf{ww}}^{k} = \Sigma_{k=1}^{n} (C_{w_i w_j}^{k})$, $\mathbf{C_{xw}} := (C_{x_1\mathbf{w}}^{1}, \ldots, C_{x_n\mathbf{w}}^{n})$ and $\mathrm{D} := \mathrm{diag}(C_{x_i x_i}^{i})$.[5]

We can verify from an industry's Hicksian substitution matrix $\mathbf{C}_{\mathbf{ww}}^{k}$ whether two inputs i and j, $i \neq j$, are complements or substitutes. The associated matrix element will come out negative in the first case and take a positive sign in the second. This interpretation carries over to the elements of the aggregate substitution matrix $\mathbf{C_{ww}}$. We already note that $\mathbf{C_{ww}}$ is negative semi-definite, due to the negative semi-definiteness of each sectoral substitution matrix $\mathbf{C}_{\mathbf{ww}}^{k}$ (cf. Diewert (1982, p. 567)). Positive or negative entries to $\mathbf{C_{xw}}$, likewise, reveal that there are superior or inferior inputs, respectively, to the making of a particular good. The intensity of sectoral scale effects is measured along the diagonal of **D**. If we had assumed constant returns to scale, **D** would be a null matrix. Then (1) would correspond to (5.1) in Diewert/Woodland (1977) and (A9) in Jones/Scheinkman (1977) as well as to (6) in Chang (1979) and (48)–(49) in Woodland (1982, Section 4.7). In the present context, however, all entries along the diagonal of **D** are positive. Therefore, firstly, **D** is positive definite and, secondly, the diagonal inverse $\mathbf{D}^{-1}$ exists and is likewise positive definite.

We define $\mathbf{M} := \mathbf{C_{ww}} - \mathbf{C_{xw}}\mathbf{D}^{-1}\mathbf{C'_{xw}}$ (cf. Gantmacher (1990, pp. 45–46)) and require that $\det(\mathbf{M}) \neq 0$. Certainly some technologies are excluded by this assumption. For example, suppose that no sector permits input substitution such that $\mathbf{C_{ww}}$ is a null matrix. Also suppose that the cost structures of two or more sectors are even identical, in which case $\mathbf{C_{xw}}\mathbf{D}^{-1}\mathbf{C'_{xw}}$ will not have full rank. Then, since $\mathbf{M} = -\mathbf{C_{xw}}\mathbf{D}^{-1}\mathbf{C'_{xw}}$, the assumed regularity of **M** will be violated. Basically, however, our regularity assumption only eliminates borderline singular cases of little economic significance. Given the existence of $\mathbf{D}^{-1}$, the regularity of **M** is necessary and sufficient for a unique solution of (1) in terms of changes in factor rents **w** and sector outputs **x**.[6] In all, due to $\det(\mathbf{M}) \neq 0$, $\mathbf{M}^{-1}$ exists and the following comparative-statics responses to changes in the world market prices **p** and national endowments **v** result (cf. Hadley (1974, pp. 107–109)):

$$(2)\quad \begin{pmatrix} d\mathbf{w} \\ d\mathbf{x} \end{pmatrix} =$$

$$\begin{pmatrix} \mathbf{M}^{-1} & -\mathbf{M}^{-1}\mathbf{C_{xw}}\mathbf{D}^{-1} \\ -\mathbf{D}^{-1}\mathbf{C'_{xw}}\mathbf{M}^{-1} & \mathbf{D}^{-1} + \mathbf{D}^{-1}\mathbf{C'_{xw}}\mathbf{M}^{-1}\mathbf{C_{xw}}\mathbf{D}^{-1} \end{pmatrix} \begin{pmatrix} d\mathbf{v} \\ d\mathbf{p} \end{pmatrix}.$$

Consequently:

$$(3)\quad \frac{\partial \mathbf{x}}{\partial \mathbf{v}} = -\mathbf{D}^{-1}\mathbf{C'_{xw}}\mathbf{M}^{-1}.$$

Recall that $\mathbf{D}^{-1}$ has a positive diagonal (with elements $1/C_{x_i x_i}^{i}$). Therefore, the respective Rybczynski effects $\partial\mathbf{x}/\partial\mathbf{v}$ will always possess the same sign as the corresponding entries to $-\mathbf{C'_{xw}}\mathbf{M}^{-1}$. Formally, introducing as **O** the null matrix of appropriate dimension:

Proposition 1 All Rybczynski derivatives $\partial\mathbf{x}/\partial\mathbf{v}$ will come out positive if, and only if, $-\mathbf{C'_{xw}}\mathbf{M}^{-1} > \mathbf{O}$.[7]

This proposition provides a necessary and sufficient condition for an increase in all sector outputs as a response to an increase in any single factor supply. The underlying intuition is that scale disadvantages on the sectoral level generate increasing opportunity costs in the economy's output space. From a normative viewpoint, then, (incomplete) specialization strategies in favor of selected industries are less rewarding, provided that inputs can be re-allocated across sectors, i.e., "if [...] the isoquants are relatively flat" (Ylönen (1987, p. 216)). Ex-

[5] *Subscripts attached to C again represent partial derivatives.*

[6] *In the terminology of the implicit function theorem, the Jacobian determinant* $\begin{vmatrix} \mathbf{C_{ww}} & \mathbf{C_{xw}} \\ \mathbf{C'_{xw}} & \mathbf{D} \end{vmatrix}$ *will be nonzero. Both* **w** *and* **x** *can then be viewed as local functions of* (**v**, **p**), *i.e. functions which are defined in some neighborhood of the economy's equilibrium state.*

[7] *All matrix and vector inequalities shall hold elementwise.*

amples highlighting the range of this result are offered in the next section. Any such example must allow for input substitution in at least one production sector, as we shall now prove. We will make use of the following observation:

Lemma 1 If no technology supports input substitution, then $\mathbf{C_{xw}} \geq \mathbf{O}$.

Proof Consider the cost function $C^i(\mathbf{w}, x_i)$ of an arbitrary sector i. By definition of this function and because of Shephard's Lemma, $C^i(\mathbf{w}, x_i) = \Sigma_j w_j C^i_{w_j}(\mathbf{w}, x_i)$ for all positive factor prices and outputs. Hence, $C^i_{x_i}(\mathbf{w}, x_i) = \Sigma_j w_j C^i_{w_j x_i}(\mathbf{w}, x_i)$. As input substitution is not permitted, the factor demands $C^i_{w_j}(\mathbf{w}, x_i)$ and, hence, the second-order derivatives $C^i_{w_j x_i}(\mathbf{w}, x_i)$ are all independent of $\mathbf{w}$. Now suppose, per absurdum, that $C_{w_k x_i}(\mathbf{w}, x_i) < 0$ for some input k. Then, for every w_k that is sufficiently large, we would obtain $C^i_{x_i}(\mathbf{w}, x_i) < 0$, which contradicts the fact that marginal cost is always positive. Finally, notice that $C_{x_i w_k}(\mathbf{w}, x_i) = C_{w_k x_i}(\mathbf{w}, x_i)$ according to Young's Theorem. □

Remark 1 Suppose that no technology supports input substitution. Then the matrix inequality $-\mathbf{C}'_{\mathbf{xw}}\mathbf{M}^{-1} > \mathbf{O}$ cannot be satisfied.

Proof As no technology permits input substitution, $\mathbf{C}^i_{\mathbf{ww}} = \mathbf{O}$ for all sectors i. Hence, by definition, $\mathbf{C_{ww}} = \mathbf{O}$. This implies $\mathbf{M} = -\mathbf{C_{xw}}\mathbf{D}^{-1}\mathbf{C}'_{\mathbf{xw}}$. Now let $\mathbf{E} := (e_{ij})$ denote the $m \times m$ identity matrix such that $\mathbf{E} = \mathbf{M}\mathbf{M}^{-1} = \mathbf{C_{xw}}\mathbf{D}^{-1}(-\mathbf{C}'_{\mathbf{xw}}\mathbf{M}^{-1})$, where $\mathbf{C_{xw}} \geq \mathbf{O}$ by Lemma 1. Therefore, if all entries to the matrix product in brackets were positive, $e_{ij} = 0$ $(i \neq j)$ would require that the entire row i of $\mathbf{C_{xw}}$ vanishes, contradicting $e_{ii} = 1$. Consequently, not all elements of $-\mathbf{C}'_{\mathbf{xw}}\mathbf{M}^{-1}$ can be positive at the same time. □

Our remark confirms that scale effects do not matter, i.e., the standard Rybczynski outcome persists, if input substitution is ruled out in all industries. In this case, expanding the economy's outputs altogether would require more of *every*, and not just one, input.

We now briefly address the adjustments in factor prices. In the standard Heckscher-Ohlin framework of two commodities and factors with constant returns to scale, the price of the factor of which an extra supply has become available will decrease whereas the other factor price will increase (cf. Woodland (1982, p. 79)). In our model economy, factor price responses to endowment changes are given by $\mathbf{M}^{-1}$ (cf. equation (2)). By definition, $\mathbf{M}$ (and hence $\mathbf{M}^{-1}$) is negative definite and will thus always possess a negative diagonal.[8] Together with the examples in the following section this proves:

Proposition 2 The own-price effects of arbitrary supply shocks are normal, whereas for $i \neq j$ the adjustments $\partial w_i/\partial v_j$ may be of either sign.

In particular, and different from both the standard set-up and the Hansson/Lundahl (1983) approach with homogeneous production functions, all factor prices may decrease as any single endowment increases. A definite statement can be made in the special case of an economy that produces its outputs from just two inputs. This statement is based on the occurrence of inferior factors like, e.g., low-skilled labor:

Remark 2 Suppose there are two inputs to production in all of the economy. Also suppose that at least one industry uses an inferior factor. Then, it follows from the inequality $-\mathbf{C}'_{\mathbf{xw}}\mathbf{M}^{-1} > \mathbf{O}$ that the factor price responses $\partial w_i/\partial v_j$ $(i \neq j)$ cannot be normal.

Proof To begin with, introduce as $\mathbf{e}$ the first n-dimensional unit vector, i.e. $\mathbf{e}' = (1\ 0\ \ldots\ 0)$. Assume that $-\mathbf{C}'_{\mathbf{xw}}\mathbf{M}^{-1} > \mathbf{O}$, and hence $-\mathbf{M}^{-1}\mathbf{C_{xw}} > \mathbf{O}$ for the product transpose. Thereby, $\mathbf{M}^{-1}\mathbf{b} > \mathbf{o}$, where $\mathbf{b} := -\mathbf{C_{xw}}\mathbf{e}$ holds the negative of the first column of $\mathbf{C_{xw}}$ (and $\mathbf{o}$ stands for the null vector of length 2). Without loss of generality, suppose that this column's first entry is negative, indicating an inferior input in sector one. Hence, $b_1 > 0$, while $b_2 < 0$. At this point, recall that $\mathbf{M}^{-1}$ is negative definite and thus has a neg-

[8] *Note that* $\mathbf{r}'\mathbf{M}\mathbf{r} = \mathbf{r}'\mathbf{C_{ww}}\mathbf{r} - \mathbf{r}'\mathbf{C_{xw}}\mathbf{D}^{-1}\mathbf{C}'_{\mathbf{xw}}\mathbf{r} \leq 0$ *for all* $\mathbf{r} \neq \mathbf{o}$ *of length m, since* $\mathbf{C_{ww}}$ *is negative semi-definite and* $\mathbf{D}^{-1}$ *is positive definite (with* $\mathbf{C}'_{\mathbf{xw}}\mathbf{r}$ *possibly equal to the null vector). Hence,* $\mathbf{r}'\mathbf{M}\mathbf{r} < 0$ *by the assumed regularity of* $\mathbf{M}$. *(Cf. Diewert/Woodland (1977, p. 392) for a similar argument in the context of constant returns to scale.)*

ative diagonal. Consequently, we conclude from $\mathbf{M}^{-1}\mathbf{b} > \mathbf{o}$ that the second element in the first row of $\mathbf{M}^{-1}$ takes a negative value (and so does the first entry to the second row by the symmetry of $\mathbf{M}^{-1}$). □

Proposition 2 and, in particular, Remark 2 are less surprising if residual profits are accounted for. As a consequence of decreasing returns and marginal cost pricing, the economy can generate positive profits. These profits also can be maintained since we assume a given number of firms per industry, e.g., when low-level market entry may not cover average cost. Then, profits possibly rise in each single industry if a factor supply shifts upwards:

Corollary 1 Consider again the n × 2 economy presented in Remark 2. Hence, suppose that both factor prices decrease in response to an expansion of a single endowment. Then all sectors will enjoy higher profits.

Proof Let $\Pi^i(p_i, \mathbf{w}) := \max_{x_i>0} \{p_i x_i - C^i(\mathbf{w}, x_i)\}$ denote the profit function of an arbitrary sector i. By Hotelling's Lemma, $\Pi^i_{w_j}(p_i, \mathbf{w}) = -C^i_{w_j}(\mathbf{w}, x_i)$ for both factor prices ($j = 1, 2$). Total differentiation of $\Pi^i(p_i, \mathbf{w})$ with respect to $\mathbf{w}$ thus gives $d\Pi^i(p_i, \mathbf{w}) = -C^i_{w_1}(\mathbf{w}, x_i)\, dw_1 - C^i_{w_2}(\mathbf{w}, x_i)\, dw_2$. As at least one of the two factor demands $C^i_{w_j}(\mathbf{w}, x_i)$ in sector i must be strictly positive, we conclude that the profit of sector i increases when both factor prices fall. □

It is important to note that the division of profits depends on firm ownership which we have not modeled, as we did not include a domestic or international capital market. Therefore, the consequences of profits on the distribution of incomes within the economy and, potentially, outside must remain indeterminate. However, given that world-market output prices are considered as exogenous, the economy's Rybczynski derivatives will not be affected. In view of the standard reciprocity relation (cf. Samuelson (1953/1954)), one might presume that non-Rybczynski responses of outputs to endowment changes always go along with non-Stolper-Samuelson effects of output price variations on factor prices. As to these effects, however, the distribution of residual incomes, maybe to omitted factors, becomes an essential issue. Yet, a complete exploration of the comparative statics associated with output price variations and, in particular, of the Stolper-Samuelson derivatives under decreasing returns to scale is beyond the scope of the present paper.

3. Computational Examples

We now present two computational examples of technologies and related cost functions, respectively, which shall further illustrate our results under different scenarios. Our first example is that of a two-sector economy with two inputs. The sectoral cost functions correspond to a production process with a fixed factor intensity in one sector and to a substitutional technology in the other. We consider different equilibrium states in which one input can also be inferior. In all three states, all Rybczynski derivatives come out positive, along with both standard and non-standard factor price responses.

We then assume, in our second example, that sector 2 employs a third input that is specific to this sector. We consider two different parameterizations of the sectoral cost function which illustrate the occurrence of complementary and inferior inputs. Once again, all Rybczynski derivatives are positive. In comparison, a standard Ricardo-Viner two-sector model with sector-specific inputs and constant returns (cf. Jones (1971)) predicts that a rise in the supply of a specific factor in one sector will increase this sector's equilibrium output, while the other sector will shrink. Also in more general models, where strictly sector-specific factors are replaced by mobile so-called 'extreme' inputs, not all Rybczynski derivatives can be positive at the same time (cf. Ruffin (1981) and Jones/Easton (1983)). In our second example, again, not every factor price response is normal.

Example 1 Let $n = m = 2$ and consider both the cost function $C^1(w_1, w_2, x_1) = (0.1w_1 + 0.05w_2)\, x_1^{1.26}$, which comes from a production process with a fixed factor intensity, as well as the cost function $C^2(w_1, w_2, x_2) = w_1 x_2^{1.2} + 2\sqrt{w_1 w_2}\, x_2^{1.1} - w_2 x_2^{1.3}$, which belongs to a substitutional technology, over some domain in the neighborhood of the economy's assumed equilibrium state $w_1 = 1.5$, $w_2 = 1$ and $x_1 = 1$, $x_2 = \eta$. Some lengthy calculations (to be obtained from the authors upon request) lead essentially to the subsequent results, depending on the level of η:

1. Let $\eta = 1.0$. We find:

$$\mathbf{C_{ww}} = \begin{pmatrix} -0.272 & 0.408 \\ 0.408 & -0.612 \end{pmatrix}, \quad \mathbf{C_{xw}} = \begin{pmatrix} 0.126 & 2.098 \\ 0.063 & 0.047 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 0.066 & 0 \\ 0 & 0.239 \end{pmatrix},$$

$$\mathbf{M} = \begin{pmatrix} -18.900 & -0.127 \\ -0.127 & -0.682 \end{pmatrix}, \quad \mathbf{M}^{-1} = \begin{pmatrix} -0.053 & 0.010 \\ 0.010 & -1.468 \end{pmatrix}, \quad -\mathbf{C'_{xw}}\mathbf{M}^{-1} = \begin{pmatrix} 0.006 & 0.091 \\ 0.111 & 0.049 \end{pmatrix}.$$

2. Let $\eta = 1.15$. Then:

$$\mathbf{C_{ww}} = \begin{pmatrix} -0.317 & 0.476 \\ 0.476 & -0.714 \end{pmatrix}, \quad \mathbf{C_{xw}} = \begin{pmatrix} 0.126 & 2.145 \\ 0.063 & 0.011 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 0.066 & 0 \\ 0 & 0.206 \end{pmatrix},$$

$$\mathbf{M} = \begin{pmatrix} -22.906 & 0.245 \\ 0.245 & -0.775 \end{pmatrix}, \quad \mathbf{M}^{-1} = \begin{pmatrix} -0.044 & -0.014 \\ -0.014 & -1.294 \end{pmatrix}, \quad -\mathbf{C'_{xw}}\mathbf{M}^{-1} = \begin{pmatrix} 0.006 & 0.083 \\ 0.094 & 0.043 \end{pmatrix}.$$

3. Let $\eta = 1.5$. This gives:

$$\mathbf{C_{ww}} = \begin{pmatrix} -0.425 & 0.638 \\ 0.638 & -0.957 \end{pmatrix}, \quad \mathbf{C_{xw}} = \begin{pmatrix} 0.126 & 2.237 \\ 0.063 & -0.065 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 0.066 & 0 \\ 0 & 0.154 \end{pmatrix},$$

$$\mathbf{M} = \begin{pmatrix} -33.215 & 1.465 \\ 1.465 & -1.045 \end{pmatrix}, \quad \mathbf{M}^{-1} = \begin{pmatrix} -0.032 & -0.045 \\ -0.045 & -1.020 \end{pmatrix}, \quad -\mathbf{C'_{xw}}\mathbf{M}^{-1} = \begin{pmatrix} 0.007 & 0.070 \\ 0.069 & 0.034 \end{pmatrix}.$$

In all three cases, every entry to $-\mathbf{C'_{xw}}\mathbf{M}^{-1}$ is positive. Consequently, $\partial x_i/\partial v_j$ is positive for all $i,j = 1, 2$. In other words, an increase in *any single* factor endowment implies an increase in *all* sector outputs, i.e., *no* sector will shrink. At the same time, in the first case, both factors are superior and the matrix $\mathbf{M}^{-1}$ exhibits the standard factor price responses. In the second case, the superiority of inputs is maintained. However, all factor prices will now decrease since all elements of $\mathbf{M}^{-1}$ are negative (cf. Proposition 2). This holds again in the third case, where factor 2 is inferior in the second industry (cf. Remark 2).

Example 2 This is an outgrowth of our first example. We take $n = 2$, $m = 3$. The first sector shall only use factors 1 and 2 and shall thereby once more possess the same technology and cost function, respectively, as before. However, now suppose that the second sector employs a specific third input such that sectoral minimum cost has an extended local representation of the form $C^2(w_1, w_2, w_3, x_2) = w_1 x_2^{1.2} + 2\sqrt{w_1 w_2}\, x_2^{1.1} - w_2 x_2^{1.3} + (\sigma\sqrt{w_1 w_2} + \tau\sqrt{w_2 w_3} + \psi w_3)\, x_2^{1.1}$. The economy's equilibrium position shall be attained at $w_1 = 1.5$, $w_2 = 1$, $w_3 = 1.5$ and $x_1 = 1$, $x_2 = 1.5$. The following are interesting parameterizations of σ, τ and ψ (computational details can be acquired from the authors):

1. Let $(\sigma, \tau, \psi) = (-0.1, 1.5, -0.54)$. We get:

$$\mathbf{C_{ww}} = \begin{pmatrix} -0.399 & 0.638 & -0.026 \\ 0.638 & -1.674 & 0.478 \\ -0.026 & 0.478 & -0.293 \end{pmatrix}, \quad \mathbf{C_{xw}} = \begin{pmatrix} 0.126 & 2.179 \\ 0.063 & 0.987 \\ 0 & 0.026 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 0.066 & 0 \\ 0 & 0.221 \end{pmatrix},$$

$$\mathbf{M} = \begin{pmatrix} -22.164 & -9.231 & -0.279 \\ -9.231 & -6.149 & 0.364 \\ -0.279 & 0.364 & -0.296 \end{pmatrix}, \quad \mathbf{M}^{-1} = \begin{pmatrix} -0.172 & 0.288 & 0.516 \\ 0.288 & -0.659 & -1.083 \\ 0.516 & -1.083 & -5.199 \end{pmatrix},$$

$$-\mathbf{C}'_{\mathbf{xw}}\mathbf{M}^{-1} = \begin{pmatrix} 0.003 & 0.005 & 0.003 \\ 0.076 & 0.050 & 0.076 \end{pmatrix}.$$

2. Let $(\sigma, \tau, \psi) = (-0.01, 0.07, -0.02)$. Accordingly:

$$\mathbf{C}_{\mathbf{ww}} = \begin{pmatrix} -0.423 & 0.638 & -0.003 \\ 0.638 & -0.990 & 0.022 \\ -0.003 & 0.022 & -0.012 \end{pmatrix}, \quad \mathbf{C}_{\mathbf{xw}} = \begin{pmatrix} 0.126 & 2.231 \\ 0.063 & -0.016 \\ 0 & 0.004 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 0.066 & 0 \\ 0 & 0.157 \end{pmatrix},$$

$$\mathbf{M} = \begin{pmatrix} -32.404 & 0.745 & -0.061 \\ 0.745 & -1.052 & 0.023 \\ -0.061 & 0.023 & -0.012 \end{pmatrix}, \quad \mathbf{M}^{-1} = \begin{pmatrix} -0.032 & -0.020 & 0.119 \\ -0.020 & -1.002 & -1.743 \\ 0.119 & -1.743 & -84.536 \end{pmatrix},$$

$$-\mathbf{C}'_{\mathbf{xw}}\mathbf{M}^{-1} = \begin{pmatrix} 0.005 & 0.066 & 0.095 \\ 0.070 & 0.035 & 0.053 \end{pmatrix}.$$

We find once more again that $-\mathbf{C}'_{\mathbf{xw}}\mathbf{M}^{-1}$ is element-wise positive. In both cases, factors 1 and 3 are complementary inputs to the second industry 2 as indicated by the respective negative off-diagonal entries to $\mathbf{C}_{\mathbf{ww}}$. (Recall from our notation in Section 2 that $\mathbf{C}_{\mathbf{ww}} = \mathbf{C}^1_{\mathbf{ww}} + \mathbf{C}^2_{\mathbf{ww}}$ and observe that $\mathbf{C}^1_{\mathbf{ww}}$ vanishes.) The second case illustrates that the simultaneous occurence of an inferior input 2 in the same sector is possible if there are more than two factors. In both cases, not all factor price responses are normal.

4. *Stability*

The above cases, simple as they are, demonstrate clearly that non-Rybczynski outcomes may emerge in a standard 2 × 2-setting if increasing marginal costs are assumed. According to Samuelson's correspondence principle (cf. Samuelson (1947)), however, such comparative-statics results are meaningful only if stability of the equilibrium is guaranteed. The argument, so-called comparative-statics paradoxes were usually associated with unstable equilibria and would otherwise vanish, has been put forward repeatedly in the literature on international trade (e.g. Mayer (1974), Ide/Takayama (1988, 1990)). For this reason, it is important to note that in our model equilibrium factor prices and outputs are *always* (locally) stable once the market-clearing process operates according to some plain, yet intuitively appealing, principles.

We suggest two standard types of adjustment processes in the factor and commodity markets, respectively (cf. Mayer (1974)). To begin with, we propose that rent variations are proportional by positive scalars k_j to the amount of excess input demand. This Walrasian price-tâtonnement shall be complemented in the goods markets by a Marshallian adjustment mechanism: outputs evolve in the direction of the difference between world demand prices and domestic supply prices (in terms of marginal cost of production), again by some positive factors of proportionality l_i. Assuming for simplicity that $k_j = l_i = 1$ for all j and i, we obtain:[9]

$$(4) \quad \dot{w}_j = \sum_{i=1}^{n} C^i_{w_j}(\mathbf{w}, x_i) - v_j \quad \text{for all } j\,,$$

$$(5) \quad \dot{x}_i = p_i - C^i_{x_i}(\mathbf{w}, x_i) \quad \text{for all } i\,,$$

where Shephard's Lemma has been used in (4).

[9] *The dot '·' indicates total differentiation with respect to time.*

Associated with the adjustment model (4)–(5) is the following linear approximation system (cf. Takayama (1985, p. 311)) which has been evaluated around an equilibrium state ($\mathbf{w}^*$, $\mathbf{x}^*$):

$$(6)\quad \begin{pmatrix} \dot{\mathbf{w}} \\ \dot{\mathbf{x}} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \mathbf{w}-\mathbf{w}^* \\ \mathbf{x}-\mathbf{x}^* \end{pmatrix}$$

with the Jacobian matrix $\mathbf{A}$ defined as:

$$(7)\quad \mathbf{A} := \begin{pmatrix} \mathbf{C_{ww}} & \mathbf{C_{xw}} \\ -\mathbf{C'_{xw}} & -\mathbf{D} \end{pmatrix}.$$

We are now prepared to introduce our next proposition:

Proposition 3 All eigenvalues of $\mathbf{A}$ are negative real numbers. The equilibrium state ($\mathbf{w}^*$, $\mathbf{x}^*$) is stable.

Proof Let λ and $\mathbf{z}$ denote an eigenvalue and related eigenvector, respectively, of $\mathbf{A}$. Hence, $\mathbf{Az} = \lambda\mathbf{z}$. Furthermore, partition $\mathbf{z}$ into two vectors $\mathbf{z}_1$ and $\mathbf{z}_2$ of suitable length such that

$$\begin{aligned}(8)\quad \mathbf{z}'\mathbf{Az} &= (\mathbf{z}_1'\mathbf{z}_2') \begin{pmatrix} \mathbf{C_{ww}} & \mathbf{C_{xw}} \\ -\mathbf{C'_{xw}} & -\mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{pmatrix} \\ &= \mathbf{z}_1'\mathbf{C_{ww}}\mathbf{z}_1 - \mathbf{z}_2'\mathbf{D}\mathbf{z}_2 \\ &= \lambda\,\mathbf{z}'\mathbf{z}\,.\end{aligned}$$

Note that $\mathbf{C_{ww}}$ is a negative semi-definite matrix (cf. Diewert (1982, p. 567)). Also recall that $\mathbf{D}$ is positive definite. Therefore, $-\mathbf{D}$ will be negative definite, and we may conclude from (8) that all eigenvalues of $\mathbf{A}$ must be real and non-positive. Regularity of $\mathbf{D}$ implies $\det(-\mathbf{D}) \neq 0$, while $\det(\mathbf{M}) \neq 0$ by assumption (cf. Section 2). Therefore

(9)

$$\begin{aligned}\det(\mathbf{A}) &= \det(-\mathbf{D})\,\det(\mathbf{C_{ww}} - \mathbf{C_{xw}}\,\mathbf{D}^{-1}\,\mathbf{C'_{xw}}) \\ &= \det(-\mathbf{D})\,\det(\mathbf{M})\end{aligned}$$

(cf. Gantmacher (1990, pp. 45–46)) requires $\det(\mathbf{A}) \neq 0$. Consequently, zero can not be an eigenvalue of $\mathbf{A}$. Thus all eigenvalues of $\mathbf{A}$ must be negative reals which is sufficient for ($\mathbf{x}^*$, $\mathbf{w}^*$) to be stable (see Takayama (1985, p. 310)). □

We thus find, unlike Mayer (1974) and Ide/Takayama (1988, 1990), that (seemingly) paradoxical comparative-statics effects may be observed even in well-behaved settings with locally stable equilibria. The following differences in the approach taken by these authors on the one hand and our model on the other hand appear to be crucial. Firstly, we assume that prices equal *marginal* rather than *average* costs and that firms operate at least at their efficient scale without loss. Secondly, Mayer and Ide/Takayama impose several restrictions upon the equilibrium outcomes of their models, following Jones (1968). They thereby neglect inferior inputs and also require that an increase in any factor price must raise the average *equilibrium* production cost of each commodity. Therefore, certain non-obvious cases of technologies and endowment changes are excluded from their approach. According to our results, the cases allowing for non-standard Rybczynski responses of industry outputs in otherwise stable equilibria must be among them.

5. *External Scale Effects*

The recent literature on total factor productivity provides empirical evidence for the existence of scale effects in terms of knowledge spillovers between production sectors and in terms of other production externalities which are not within the direct exploitation potential of a single firm or industry (e.g., Caballero/Lyons (1990), Benarroch (1997), Midelfart/Steen (1999)). In this context, a firm which is subject to increasing marginal cost appears to operate under constant or even increasing returns to scale if there are inter-industry shifts in productivity. We now show that our results are robust with respect to a simple class of production externalities. Our analysis once more sticks as closely as possible to the static model of perfect competition.

We start from the notion that the output x_i of each arbitrary industry i is a function $f_i(\cdot, E, V)$ of several input factors and of proxies E and V of activity-based externalities and technical progress, respectively (Caballero/Lyons (1990)). As E and V are by assumption out of control of industry i, their respective output elasticities can be locally normalized to equal unity (cf. Caballero/Lyons (1990, pp. 808–809), Basu/Fernald (1995, p. 169) for a formal exposition), in which case the industry production function has a local representation of the form $x_i = \alpha h_i(\cdot)$, where $\alpha := EV$. From the viewpoint of a single firm or industry, increases in α are perceived as exogenous downward shifts of the (upward-sloping) marginal cost curve. We assume that α is a non-decreasing function of the economy's aggregate economic activity as measured by the levels of total endowments. Recall that these endowments are at full employment by assumption. They can thus serve as a means of transmission of global knowledge spillovers and localized agglomeration forces across industries. In the literature on new economic geography and on endogenous economic growth, this role is often attributed to the stock of physical capital (Romer (1986)) and to the 'human capital' or skill of a typical worker (Lucas (1988)). Then, $\lambda x_i = h(\cdot)$ with $\lambda := 1/\alpha$. Hence, λ decreases if external scale advantages have a positive effect on the total factor productivity in sector i. For simplicity, we pretend that this effect is the same in every industry.

Now let $y_i := \lambda x_i$ for all industries i. Also, assume without loss of generality that $\lambda = 1$ in the economy's initial equilibrium. With this change in notation, our results in Sections 2–4 can all be retained. Furthermore:

Proposition 4 The signs of all positive Rybczynski derivatives are preserved under a decrease of λ.

Proof Suppose there is an extra supply of resources which generates an aggregate scale effect $d\lambda < 0$. By $dy_i = dx_i + x_i d\lambda$ and $x_i > 0$, we conclude that $dx_i > 0$ whenever $dy_i > 0$. Hence, the signs of all positive derivatives $\partial y_i/\partial v_j$ carry over to $\partial x_i/\partial v_j$. □

It even turns out that there is more room for a positive output response in all industries to an expansion of endowments, since output x_i will also increase if $dy_i \leq 0$ as long as $|dy_i| < x_i|d\lambda|$. Again, we think that there is nothing 'paradoxical' about this result. In contrast, our findings are still intuitively plausible and consistent with a profit-maximizing behavior of firms or industries in a competitive environment.

6. Conclusion

The purpose of this paper was to examine the comparative statics of endowment changes in a small open and competitive economy with a finite number of goods and factors. We assumed that this economy is exposed to diseconomies of scale, and thus to increasing marginal costs, within each industry. This assumption was motivated by a critical assessment of the standard replication argument for constant returns, both from a theoretical and an empirical perspective.

Scale disadvantages create increasing opportunity costs in the economy's output space. From a normative viewpoint, then, (incomplete) specialization strategies in favor of selected industries are less rewarding, provided that inputs can be re-allocated across sectors. Common intuition thus suggests that all production sectors should benefit from an extra supply of any scarce resource. We proved as our major result that this conjecture, although in conflict with Rybczynski's theorem, can hold true, indeed, if the production technologies permit input substitution. In contrast to the literature, we showed that our result is fully compatible with the Walrasian and Marshallian stability of the underlying equilibrium. Our findings also turned out to be robust with respect to global shifts in total factor productivity which raise the economy's outputs beyond private returns.

As the Rybczynski theorem was developed in the context of the Heckscher-Ohlin or factor proportions theory of international trade, empirical tests of this theory in the literature may shed some light on the relevance of the non-Rybczynski responses we have derived. A seminal book is Leamer (1984). Alas, he argues that it is "... wildly optimistic to recover the Rybczynski coefficients from data on trade and endowments by a regression of trade on a subset of excess factor supplies" (Leamer (1984, p. 159)). Therefore, he opts for an estimation of a reduced-form model. Unfortunately, estimates of the Rybczynski coefficients cannot be easily constructed from this model. However, Harrigan (1995), who builds on Leamer's work (cf. also the more recent contribution by Feenstra (2004)), concludes from a Heckscher-Ohlin-Vanek version of the factor proportions model that differences in output levels across countries can be explained linearly by differences in national factor endowments. His estimation results indicate that capital has a positive equilibrium effect on all ten outputs considered (cf. Harrigan (1995, Table 4, pp. 135–136)). Thus, we find a non-Rybczynski output response with respect to an important input. This demonstrates at least that the outcome of our model is more than just a theoretical fiction.

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