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RATIONALITY OF LIMITED RATIONALITY: SOME AGGREGATE IMPLICATIONS*

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In this paper we let economic agents choose whether to become fully rational or stay boundedly rational. Boundedly rational agents are less sophisticated in their information processing abilities. It is costly to acquire information needed to become fully rational, and thus not all agents are willing to incur those costs. We then explore the aggregate effects of endogenizing the decision whether the agent should or should not become fully rational in handling information. Since fully and boundedly rational individuals make different saving and consumption decisions, their interaction has a direct impact on the size of capital stock, aggregate savings and the volatility of output. E.g. we are able to show that in many circumstances our model will imply a smoother consumption than would be observed in a model with only fully rational consumers. (JEL D81, E21, E22).

1. Introduction

In recent years there has been an increase in interest in the study of models where economic agents have diverse information as well as different information processing capabilities. Models with only fully informed persons fail to take into account the implications of the diversity of individual behavior and ability. Many economists since Simon's seminal work on bounded rationality (Simon, 1972) have recognized that actual behavior in many markets shows significant deviations from the predictions of models with only fully informed people. For example, many studies have shown that the behavior of aggregate consumption is very difficult to explain using the permanent income theory, i.e. a model with only fully rational consumers (see e.g.

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Mankiw, 1981, Campbell and Deaton, 1989 and Gali, 1990). Aggregate consumption appears to fluctuate less than what is predicted by the permanent income theory. In fact, Campbell and Mankiw, 1989, have shown that the behavior of U.S. aggregate consumption can be explained better by a model where there are a mix of consumers, fully rational consumers and consumers who follow a »rule of thumb» in their consumption behavior. In addition, the work by Shiller (collected in Shiller, 1989) shows that the behavior of the stock, bond, and real estate markets are hard to understand when one relies on the explanations of models that use only fully rational economic agents.

Here we let agents choose whether to acquire the costly information needed to become fully informed. Because the less-sophisticated agents are less able to process information, we assume that they solve a much simpler decision problem where they maximize the sum of the utility from today's consumption and the discounted value of the utility from tomorrow's expected consumption. This, in fact, means that they are using the rule of thumb which is a point estimate of future interest rates. The point estimate initially is assumed to be a certainty equivalent of the future interest rates. This may not be an unreasonable decision rule since, as argued by Akerlof and Yellen, 1985a, 1985b, the cost to the individual of using this type of a rule as opposed to fully optimizing is small. Also, there is evidence from cognitive psychology that people have a tendency to underestimate uncertainty in their decision making (e.g. Arrow, 1982, discusses this evidence and indicates how the use of heuristics as found in the work of cognitive psychologists, see for example, Kahneman, Slovic, and Tversky, 1982, results in this type of underestimation). For simplicity, we study the polar case where consumers disregard uncertainty. We indicate, however, that our results are robust to the assumption that consumers underestimate uncertainty only slightly. Hey, 1981, in a somewhat different framework argues that if the decision problem is sufficiently complex (so that the computational costs are not insignificant) it would not be unreasonable for the agent to adopt a rule of thumb as his criterion. The case where individuals are overly optimistic or pessimistic and the weights (or probabilities of the future states) are incorrect will also be explored to indicate how optimism or pessimism affect the results.

Since fully and boundedly rational individuals make different saving and consumption decisions, their interaction does have a direct impact on the size of the capital stock, aggregate savings, and the volatility of output. We are also able to show that in many circumstances (depending on the size of the relative risk aversion) our model will have smoother consumption than would be observed in a model with only fully rational consumers. In addition, government policies that affect information availability or changes that affect the degree of uncertainty facing an economy will alter the proportion of the two types of agents. Thus, the purpose of this paper is to explore the aggregate effects of endogenizing the decision of whether one should or should not become fully rational in handling information.

Haltiwanger and Waldman, 1985, 1991, in a model where the proportions of the different types of agents are fixed, ask under what conditions either fully rational agents or naive agents have a disproportionately large effect on equilibrium. The distinction between smart and naive agents is in their abilities to form expectations. Smart agents have rational expectations, and naive agents have incorrect expectations, i.e., they use the mean of their subjective probability distribution. Possen and Puhakka, 1994, in a model very similar to the one here analyze aggregate implications when there are two types of agents. However, in that paper the proportions of the two types of agents are exogenously given.

Additional recent works in this area include Russell and Thaler, 1985, and DeLong, Shleifer, Summers, and Waldmann, 1989, 1990, 1991. Russell and Thaler also introduce two types of agents, rational and quasi-rational. Quasi-rational agents have misperceptions about the characteristics technology. DeLong, Shleifer, Summers, and Waldmann explore the effects of noise traders on the financial markets. These traders have misperceptions about the mean and the variance of the returns. They find that these traders have a significant effect on the markets.

Our paper is organized as follows. The model is introduced in the next section. The proportion of fully informed individuals in the economy is derived in section 3. In section 4 the aggregate capital stock is obtained and in section 5 the impact of changes in some of the underlying parameters is analyzed. Section 6 contains some concluding observations.

2. *The Model*

In this paper, a two-period model with uncertainty similar to the one in Possen and Puhakka, 1994, is used. The economy contains no money; rather, consumers can invest their savings in capital. In the first period people have endowments which are partly saved and partly consumed. The second period random output is produced from savings all of which is held in the form of capital. One could interpret the first period's endowment to be initial capital stock and first period output as being produced by a non-random linear technology. For simplicity, production uses only one input, capital.

Consumers decide how much to consume today and how much to save for tomorrow. It is assumed that there is a spectrum of consumers who differ only in their information gathering capabilities. If the cost of obtaining information is very low it is worthwhile for agents to gather information about all the future contingencies. Those agents who acquire precise information about all future contingencies of the economy are called fully informed consumers. Other agents are assumed to have limited information gathering abilities and thus have to pay to obtain information about all future contingencies. For these agents it is shown that they are at least as well off if they take a subjective view about uncertainty and operate with certainty equivalents for example. The idea that is being modelled here is that consumers who operate with limited information have a less discriminating view than fully informed people of what the future might entail.

In the second period there is uncertainty about the productivity of capital. It is assumed that the strictly concave technology can take two forms, $f(k)$ and $h(k)$, with the probabilities π and $1-\pi$, and with the properties $f(k) > h(k)$, and $f'(k) > h'(k)$ for all $k > 0$. Thus output in the two states will be

$$(1) \quad y_1 = f[\gamma k_o + (1-\gamma)\hat{k}_o]$$

and

$$(2) \quad y_2 = h[\gamma k_o + (1-\gamma)\hat{k}_o]$$

where γ denotes the fraction of fully informed people and k_o ($\hat{k}_o$) represents the average savings of the fully (imperfectly) informed consumers. In our previous work γ was taken to be exogenous; here, it is made endogenous. Agents are able to choose whether they want to become fully informed or not.

Since people differ in the degree to which they can acquire information, the fee required to attain the information to take all contingencies into account varies across individuals. Those people who do not find it worthwhile to pay the fee remain boundedly rational in handling information and estimate the future as an average of the possible future states. Thus, in the model it will be shown that unsophisticated consumers take the interest rate to be an average of tomorrow's rates of interest initially.

To focus on differences in information processing abilities, all individuals are assumed to have the same strictly concave periodic utility function, $u(c)$, the same endowments, w_o, and the same discount factor δ. Fully informed consumers solve the following decision problem.

$$(\text{P.1}) \quad \max_{\{c_o, c_1^1, c_1^2 \geq 0\}} \quad u(c_o) + \delta[\pi u(c_1^1) + (1-\pi)\, u(c_1^2)]$$

subject to

$$c_o + \frac{c_1^1}{R_1} \leq w_o - g$$

$$c_o + \frac{c_1^2}{R_2} \leq w_o - g$$

and $\delta = (1 + \rho)^{-1}$, where ρ is the rate of time preference. R_1 (R_2) is the gross rate of interest in state 1 (2). The savings of the fully informed person, k, satisfies $k = w_0 - c_0 - g$, and $k > 0$ if the indifference curves do not intersect the axes and $g < w_0$. One should note that g operates exactly as a lump-sum tax.

It is assumed that individuals are distributed such that the cost of acquiring the information necessary to become fully-informed, g, varies uniformly between a minimum of $\underline{g}$ and a maximum of $\bar{g}$. Furthermore, $\underline{g} \geq 0$ and $\bar{g} \leq w_0$. Substituting the budget constraints into the objective function and differentiating with respect to c_0 yields the first order condition

$$(3) \quad u'(c_0) = \delta\,[\pi R_1 u'(R_1(w_0 - c_0 - g)) + (1-\pi) R_2 u'(R_2(w_0 - c_0 - g))]$$

Consumers who decide not to acquire information solve the following decision problem[1]

$$(\text{P.2}) \quad \max_{\{\hat{c}_0, \tilde{c}_1 \geq 0\}} \; u(\hat{c}_0) + \delta u(\tilde{c}_1)$$

subject to

$$\hat{c}_0 + \frac{\tilde{c}_1}{\pi R_1 + (1-\pi) R_2} \leq w_0$$

They maximize the sum of today's utility and the discounted utility of tomorrow's expected consumption. In fact, this criterion is the same as using the rule of thumb of estimating tomorrow's rate of interest to be the average of the two possible rates of interest. Here the savings become $\hat{k}_0 = w_0 - \hat{c}_0$ and $\tilde{c}_1$ denotes a plan for future consumption. The actual consumption, however, will be

$$\hat{c}_1^1 = R_1(w_0 - \hat{c}_0) \text{ and } \hat{c}_1^2 = R_2(w_0 - \hat{c}_0) .$$

For fully informed individuals there is no difference between plans and actual consumption because R_1 and R_2 will be the equilibrium interest rates. The first-order condition for the imperfectly informed person's problem is

$$(4) \quad u'(\hat{c}_0) = \delta(\pi R_1 + (1-\pi) R_2)\, u'((\pi R_1 + (1-\pi) R_2)(w_0 - \hat{c}_0))$$

Imperfectly informed consumers behave as if they do not care about uncertainty.[2] One can ask if the less sophisticated agents are better off in an ex post sense. Their ex post discounted utility is defined here as being equal to the sum of the utility received across the periods weighted by the objective probabilities that the various states occur. We are not trying to model the process by which the choice of becoming fully informed is made; rather, we look at the outcome of the decision given the criterion so that no individual will be disappointed in an expected ex post sense. More formally, the ex post utility is defined as:

$$(5) \quad V(\hat{c}_0) = u(\hat{c}_0) + \delta\,[\pi u(R_1(w_0 - \hat{c}_0)) + (1-\pi)\, u(R_2(w_0 - \hat{c}_0))] .$$

This criterion uses actual consumption as does $V(c_0)$. Note that in (5) the expected utility is being evaluated in terms of realizations for the rule of thumb individuals. It is clear that $V(c_0) \geq V(\hat{c}_0)$ since c_0 is the level of consumption that maximizes $V(c_0)$. The next section considers the difference $V(c_0) - V(\hat{c}_0)$ when consumers must pay a fee to become fully informed.

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[1] *Expected consumption, $\tilde{c}_1$, can be defined as:*

$$\tilde{c}_1 = \tilde{c}_1^1 \pi + (1-\pi)\, \tilde{c}_1^2$$

where $\tilde{c}_1^1$ and $\tilde{c}_1^2$ are the amounts of consumption the less sophisticated agents expect to consume in period 1 state 1 and state 2, respectively. These levels of consumption face the constraints

$$\hat{c}_0 + \frac{\tilde{c}_1^1}{R_1} \leq w_0$$

$$\hat{c}_0 + \frac{\tilde{c}_1^2}{R_2} \leq w_0$$

Substituting these constraints into the definition of expected consumption yields the constraint in (P.2).

[2] *Consider the solution to (P.1) when interest rates are $R_1 - \mu$ and $R_1 + \pi/(1-\pi)\mu$ in states one and two respectively, i.e. we are considering a decision maker who understands that there is some uncertainty [note that in the resulting equilibrium $R_1 > R_2$]. One can show [c.f. Lemma 1 and 2 in Possen and Puhakka (1994)] that $dc_0/d\mu \gtreqless 0$ whenever RRA $\gtreqless 1$. This means that our results will also hold in the intermediate case where uncertainty is not completely disregarded, but rather is underestimated.*

3. The Endogenization of the Proportion of Agents

As argued earlier, to be able to make sophisticated decisions individuals have to devote resources to acquire more information or to improve their information gathering and processing abilities. Wolff and Baumol, 1989, present evidence on the explosive growth of the labor force engaged in information related activities in the United States economy during the last 25 years. Information gathering is costly but it should improve the consequences of decisions under uncertainty. Economic agents differ, however, in their costs (or abilities) of acquiring information as well as in the cost to them of becoming fully informed. Thus, it is argued that in order to become fully aware of all of the relevant information consumers must devote resources in the amount of g to gather that information. Therefore, their utility becomes:

$$(6)\quad V(c_0, g) = u(c_0) + \delta[\pi u(R_1(w_0 - c_0 - g)) + (1-\pi)\, u(R_2(w_0 - c_0 - g))]\,.$$

The issue now is to determine if there is a g^* such that $V(c_0, g^*) = V(\hat{c}_0)$ and that this makes the person with that particular g^* indifferent between remaining imperfectly informed and becoming fully informed. If there is such a g^* it also means that there is a γ^* (the fraction of the population that consists of fully informed consumers) such that $0 < \gamma^* < 1$. In the remainder of the paper it will be assumed that individuals are distributed uniformly over the range $[\underline{g}, \bar{g}]$ where $\underline{g} \geq 0$ and $\bar{g} \leq w_0$.

Proposition 1. There is a g^* in the range $\underline{g} < g^* < \bar{g}$ such that $V(c_0, g^*) = V(\hat{c}_0)$, when individuals are distributed over the whole range $[0, w_0]$. If $\underline{g} > 0$ and $\bar{g} < w_0$ it is possible to have a corner solution such that $g^* = \underline{g}$ or $g^* = \bar{g}$, i.e. $V(c_0, \underline{g}) \leq V(\hat{c}_0)$ or $V(c_0, \bar{g}) \geq V(c_0)$.

Proof: For $g = 0$ it was shown above that $V(c_0, 0) \geq V(\hat{c}_0)$ and if $c_0 \neq \hat{c}_0$ and the utility function is strictly concave it follows immediately that the relationship becomes $V(c_0, 0) > V(\hat{c}_0)$. At the other extreme if $g = w_0$ one obtains $V(\hat{c}_0) > V(0, w_0)$. To complete the proof one needs to differentiate (6) to yield:

$$\frac{\partial V(c_0, g)}{\partial g} = \{u'(c_0) - \delta[\pi R_1 u'(R_1(w_0 - c_0 - g)) + (1-\pi) R_2 u'(R_2(w_0 - c_0 - g))]\}\,\partial c_0/\partial g - \delta\{\pi R_1(u'(R_1(w_0 - c_0 - g)) + (1-\pi) R_2 u'(R_2(w_0 - c_0 - g))\} < 0\,.$$

Since the differentiation is done at the optimal level of c_0 the coefficient of $\partial c_0/\partial g$ which is just (3) equals zero, and thus the sign follows immediately. Since $V(c_0, 0) > V(\hat{c}_0)$ and $V(\hat{c}_0) > V(0, w_0)$ and $V(c_0, g)$ declines monotonically as g gets larger, there exists a g^* such that $\underline{g} < g^* < \bar{g}$ where $V(c_0, g^*) = V(\hat{c}_0)$ Q.E.D.

Figure 1 illustrates Proposition 1. What happens to g^* if the distribution of the g's is changed? Consider an increase in everybody's g. The critical value of g, g^*, remains the same since it is the solution to the equation $V(c_0, g^*) = V(\hat{c}_0)$. However, the corresponding critical value of γ, $\gamma^* = \dfrac{g^* - \underline{g}}{\bar{g} - \underline{g}}$ will change. For example, if the range changes to $[\underline{g} + \varepsilon, \bar{g} + \varepsilon]$ where $\varepsilon > 0$, the critical value of γ declines, and the number of fully informed individuals in the economy will be reduced.

4. Derivation of the Aggregate Capital Stock

In this section of the paper the size of the aggregate capital stock will be derived. To

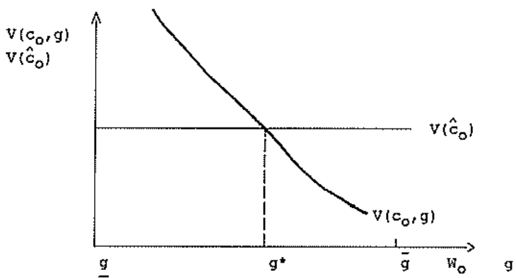

Figure 1. The Endogenous Determination of g^*.

make the analysis tractable two examples are considered. Case (a) assumes $u(c) = c^{\mu}$ and (b) assumes $u(c) = -\mu c^{-\mu}$. In case (a) the relative risk aversion (RRA) = $1 - \mu$ and in (b) RRA = $1 + \mu$. It is assumed that $\mu < 1$. For case (a) the first-order conditions (3) and (4) respectively reduce to:

$$(7) \quad \left[\frac{w_o - c_o - g}{c_o}\right]^{1-\mu} = \delta\,[\pi R_1^{\mu} + (1-\pi)\,R_2^{\mu}]$$

$$(8) \quad \left[\frac{w_o - \hat{c}_o}{\hat{c}_o}\right]^{1-\mu} = \delta\,[\pi R_1 + (1-\pi)\,R_2]^{\mu}$$

For fully informed individuals one can re-write the first-order condition as

$$\frac{c_o^i}{w_o - c_o^i - g^i} = \left[\frac{1}{\delta\,[\pi R_1^{\mu} + (1-\pi)\,R_2^{\mu}]}\right]^{\frac{1}{1-\mu}} = A$$

or $c_o^i \dfrac{(1+A)}{A} = w_o - g^i$.

The superscript i is introduced to indicate that among the fully informed individuals the fee paid to obtain the requisite information varies. In fact, fully informed individuals are distributed uniformly over the range $[g, g^*]$. One can then aggregate over all the fully informed individuals to obtain:

$$\frac{1}{\bar{g} - \underline{g}}\,\frac{(1+A)}{A}\int_{\underline{g}}^{g^*} c_o^i\,dg^i = \frac{1}{\bar{g} - \underline{g}}\int_{\underline{g}}^{g^*} (w_o - g^i)\,dg^i$$

$$= \frac{g^* - \underline{g}}{\bar{g} - \underline{g}}\left[w_o - \frac{g^* + \underline{g}}{2}\right]$$

$$= \gamma^*\left[w_o - \frac{(g^* + \underline{g})}{2}\right]$$

Since the capital stock for individual i is $k_i = w_o - c_o - g_i$, one can aggregate over all fully informed individuals to yield:

$$(9) \quad \frac{1}{\bar{g} - \underline{g}}\int_{\underline{g}}^{g^*} k_i\,dg^i = \frac{1}{\bar{g} - \underline{g}}\int_{\underline{g}}^{g^*} (w_o - c_o^i - g^i)\,dg^i$$

$$= \frac{1}{\bar{g} - \underline{g}}\left(\int_{\underline{g}}^{g^*} (w_o - g^i)\,dg^i - \int_{\underline{g}}^{g^*} c_o^i\,dg^i\right) = \frac{1}{\bar{g} - \underline{g}}\left[1 - \frac{A}{1+A}\right]\int_{\underline{g}}^{g^*} (w_o^i - g)\,dg^i$$

$$= \frac{1}{1+A}\,\gamma^*\left[w_o - \frac{(g^* + \underline{g})}{2}\right]$$

$$= \gamma^* k_o$$

For the boundedly rational agents one can re-write (8) as:

$$\frac{\hat{c}_o}{(w_o - \hat{c}_o)} = \left[\frac{1}{\delta\,[\pi R_1 + (1_1 - \pi)\,R_2]^{\mu}}\right]^{\frac{1}{1-\mu}} = B$$

or $\hat{c}_o = \dfrac{B}{1+B}\,w_o$.

There is no superscript in the above equation because all unsophisticated individuals are assumed to be alike. We have thus proven

Proposition 2. If the relative risk aversion is less than one, the aggregate capital stock for the unsophisticated agent is

$$(1 - \gamma^*)\,\hat{k}_o = (1 - \gamma^*)\,(w_o - \hat{c}_o) = (1 - \gamma^*)\,w_o\,\frac{1}{1+B}.$$

and the overall capital stock for the economy as a whole becomes:

$$\gamma^* k_o + (1 - \gamma^*)\,\hat{k}_o = \gamma^*\frac{1}{1+A}\left[w_o - \frac{(g^* + \underline{g})}{2}\right] + (1 - \gamma^*)\,\frac{1}{1+B}\,w_o.$$

Do less sophisticated agents have a larger capital stock than the fully informed individuals in this case? They will if B < A or if $[\pi R_1 + (1-\pi) R_2]^{\mu} > [\pi R_1^{\mu} + (1-\pi) R_2^{\mu}]$. However, this inequality follows from concavity. In this case RRA is less than one and, as indicated in Puhakka and Possen (1994), substitution effects dominate in savings behavior, resulting in the unsophisticated agents saving more than the fully informed. Thus, an economy that has both informed and less sophisticated agents will have a larger capital stock than one that consists only of fully informed agents. Moreover, any policy that increases g^*, and in turn γ^* in case (a) will end up reducing the aggregate amount of capital in the economy.

Proposition 3. If the relative risk aversion is greater than one, the aggregate capital stock becomes

$$\gamma^* k_o + (1-\gamma^*)\hat{k}_o =$$

$$\gamma^* \frac{1}{1+\tilde{A}}\left[w_o - \frac{(g^*+g)}{2}\right] + (1-\gamma^*)\frac{1}{1+\tilde{B}} w_o$$

$$\text{where } \tilde{A} = \left[\frac{1}{\delta\pi R_1^{-\mu} + (1-\pi) R_2^{-\mu}}\right]^{\frac{1}{\mu+1}}$$

$$\text{and } \quad \tilde{B} = \left[\frac{1}{[\pi R_1 + (1-\pi) R_2]^{-\mu}}\right]^{\frac{1}{\mu+1}}$$

Proof: The proof here uses the same type of reasoning as in Proposition 3.

The first order conditions (3) and (4) respectively for case (b) are

$$(10) \quad \left[\frac{w_o - c_o - g}{c_o}\right]^{1+\mu} = \delta[\pi R_1^{-\mu} + (1-\pi) R_2^{-\mu}]$$

$$(11) \quad \left[\frac{w_o - \hat{c}_o}{\hat{c}_o}\right]^{1+\mu} = \delta[\pi R_1 + (1-\pi) R_2]^{-\mu}$$

Since $f(x) = x^{-\mu}$ is a strictly convex function one obtains from Jensen's inequality that $\hat{c}_o > c_o$. Finally, note that for $g = g^*$, $V(c_o, g^*) = V(\hat{c}_o)$ and this equality can only hold for $\hat{c}_o > c_o$ if $w_o - c_o - g^* > w_o - \hat{c}_o$. Therefore, the savings of fully informed agents are greater than that savings of imperfectly informed ones in case (b). Thus an economy with only fully informed agents would have a larger capital stock than one that consists of both fully informed and unsophisticated consumers. This also means that when RRA > 1 aggregate consumption in the second period fluctuates less than in an economy with only fully rational consumers. Thus, it appears that the introduction of imperfectly informed rule-of-thumb consumers is potentially an important ingredient in models that try to explain the data on aggregate consumption. Moreover, one also has the result that a change that increases g^* and in turn γ^* will in this case end up raising total savings and aggregate capital stock.

Q.E.D.

Since fully informed and imperfectly informed agents save different amounts (except in the degenerate case where the utility function is linear and $\mu = 1$) their interaction has real consequences on the economy and any change that alters the proportion of the two types will affect the magnitude of the aggregate capital stock. We showed in our other paper (Possen and Puhakka, 1994) that imperfectly informed consumers have a disproportionately large effect on equilibria regardless of their attitude towards risk. Here we have shown that our earlier results are robust to having consumers endogenously decide whether they want to be fully informed or not.

5. *Impact of Changes in Parameter Values*

To focus on the impact of increased uncertainty in the economy the spread between the return in the good state, R_1, and that in the bad state, R_2, is increased while leaving the expected value of the future return unchanged. This condition can be represented as:

$$(12) \quad dR_2 = -\pi dR_1/(1-\pi)$$

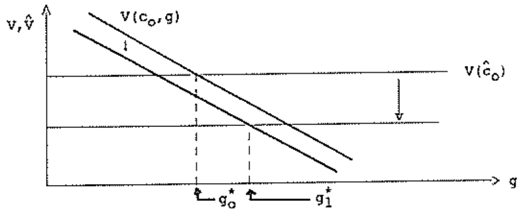

Figure 2. The Effect of an Increased Uncertainty on g^*.

where $dR_1 > 0$ and thus $dR_2 < = 0$. The effect of increased uncertainty on g^* can then be deduced by considering again the condition $V(c_o, g^*) = V(\hat{c}_o) \equiv \hat{V}$.

Proposition 4. An increase in uncertainty will increase g^* in both of the special cases (a) and (b).

Proof: See Appendix.

Figure 2 illustrates Proposition 4. In particular, it means that at a given g the expected utility for the unsophisticated will drop more than that for the fully rational agent leading to an increase in g^*. The decrease in the expected utility is due only to the direct effect of a change in uncertainty because of the envelope theorem. An increase in uncertainty does not cause adjustment in the consumption and saving behavior of the unsophisticated consumers. Why is the direct effect of an increase in uncertainty then bigger for the unsophisticated than for the sophisticated consumer? We evaluate their expected utilities by using the same function, the optimum of which is c_o, not $\hat{c}_o$. For c_o the envelope theorem clearly holds, and changes in uncertainty will lead to an optimal degree of change in the value function. However, $\hat{c}_o$ was not chosen as an optimum of the respective value function. Thus, the change in value must be bigger for $V(\hat{c}_o)$.

Proposition 4 means that the response of the unsophisticated consumers dominates that of the fully rational ones. The unsophisticated agents do not respond optimally, and thus incur a bigger utility loss.

To determine the effect on the magnitude of the capital stock for case (a) one can differentiate the first order condition (7) with respect to R_1 given that (12) holds to yield:

$$\frac{dc_o}{dR_1} = -\frac{\mu\delta\pi\,[R_1^{\mu-1} - R_2^{\mu-1}]}{(1-\mu)\left(\frac{c_o}{w_o - c_o - g}\right)^{\mu}\left(\frac{w_o - g}{c_o^2}\right)} > 0$$

Thus for a fully informed individual with a given g an increase in uncertainty increases his c_o and reduces his savings, $w_o - c_o - g$. Finally, since savings of the fully informed agents are less than the savings of the imperfectly informed ones, the savings of the fully informed agents are reduced, and the proportion of fully informed agents in the economy is increased, the aggregate capital stock in the economy is reduced.

To evaluate the impact on capital for case (b) one can, as above, differentiate first order condition (10) with respect to R_1, given (12), to yield:

$$\frac{dc_o}{dR_1} = -\frac{\mu\delta\pi\,[R_1^{-\mu-1} - R_2^{-\mu-1}]}{(1-\mu)\left(\frac{w_o - c_o - g}{c_o}\right)^{\mu}\left(\frac{w_o - g}{c_o^2}\right)} > 0$$

An increase in uncertainty reduces c_o for a fully informed person and raises his savings, $w_o - c_o - g$. Therefore, the aggregate capital stock will be increased since fully informed agents increase their savings and the proportion of fully informed individuals in the economy is increased.

Although the impact on the size of the capital stock depends on the utility function chosen, as shown in Proposition 4, in both cases the proportion of fully informed people in the economy is increased by the greater degree of uncertainty.

Thus far the unsophisticated individuals have been assumed to use the correct probabilities about the future. However, if they do not take the different states into account in their decision making, it is also quite likely that they will use incorrect probabilities in making their consumption-saving calculation. For example, if they use $\hat{\hat{\pi}} > \pi$ as their predictor one can see immediately from the first order condition (8) that in case (a) their initial period consumption $\hat{c}_o$, will be reduced, and their saving, $w - \hat{\hat{c}}_o$, increased. Moreover, by differ-

entiating equation $V(c_o, g^*) = V(\hat{c}_o)$ (from (5) and (6) for case (a)) with respect to co, $\hat{c}_o$, and g^* one can determine the impact of incorrect probabilities on g^*,
i.e.,

(13) $$[\hat{c}_o^{\mu-1} - \delta\,(w_o - \hat{c}_o)^{\mu-1}\,(\pi R_1^{\mu} + (1-\pi)\,R_2^{\mu})\,d\hat{c}_o = -\,\delta\,[\pi R_1^{\mu} + (1-\pi)\,R_2^{\mu}]\;[w_o - c_o - g^*]^{\mu-1}\,dg^*\,.$$

The coefficient of dc_o is zero since it is just the first order condition (7). Moreover, the probabilities in (6) are objective probabilities since they refer to outcomes and not plans. From (13) one sees that the coefficient of $d\hat{c}_o$ is positive. Finally, since an increase in $\hat{\pi}$ reduces $\hat{\hat{c}}_o$ below $\hat{c}_o$, the incorrect probabilities will increase g^* and in turn increase the proportion of fully informed individuals, γ^*, in the economy.

The impact of the above changes on the capital stock, unfortunately, is much harder to determine. The effect on the unsophisticated individuals acts to increase capital and one would expect this effect to dominate since that is the direct effect. However, since the proportion of unsophisticated individuals in the economy is reduced and the savings of the fully informed agents is less than that of the imperfectly informed ones it is possible for the change in the aggregate capital stock to go in the other direction.

For case (b), $\hat{\hat{\pi}} > \pi$ implies from (11) that $\hat{\hat{c}}_o$ will be increased and the savings of the imperfectly informed individuals, $w_o - \hat{\hat{c}}_o$, reduced. To determine the effect on g^* one must differentiate $V(c_o, g^*) = V(\hat{c}_o)$ with respect to $\hat{c}_o$, c_o and g^* to yield:

(14) $$\{\hat{c}_o^{-\mu-1} - \delta\,[\pi R_1^{-\mu} + (1-\pi)\,R_2^{-\mu}]\,[W_o - \hat{c}_o]^{-\mu-1}\}\,d\hat{c}_o = -\,\delta\,[\pi R_1^{-\mu} + (1-\pi)\,R_2^{-\mu}]\,[w_o - c_o - g^*]^{-\mu-1}\,dg^*$$

The coefficient of dc_o is zero since it equals the first order condition (10) and the coefficient of $d\hat{c}_o$ from (11) is negative. Therefore, an increase $\hat{\hat{\pi}}$ and in turn $d\hat{c}_o$ will result in an increase in g^* and γ^* as was true in case (a). In addition, the impact on the capital stock is ambiguous. The savings of the unsophisticated individuals is reduced but the proportion of fully informed individuals (who have larger savings) is increased.

In conclusion, a few comments about the properties of equilibria in this type of a setting are discussed. Let the proportion of fully informed people be γ^* (corresponding to g^*). Then the equilibrium conditions in the two states can be reduced to the following:

(15) $$F \equiv f\,[\gamma^* k_o(R_1, R_2) + (1-\gamma^*)\,\hat{k}_o(\bar{R})] - R_1\,[\gamma^* k_o(R_1, R_2) + (1-\gamma^*)\,\hat{k}_o(\bar{R})] = 0$$

(16) $$H \equiv h\,[\gamma^* k_o(R_1, R_2) + (1-\gamma^*)\,\hat{k}_o(\bar{R})] - R_1\,[\gamma^* k_o(R_1, R_2) + (1-\gamma^*)\,\hat{k}_o(\bar{R})] = 0$$

where $\bar{R} = \pi R_1 + (1-\pi)\,R_2$. The existence of a set of R_1 and R_2 that yields an equilibrium can be shown (see Possen and Puhakka, 1994). For example, when RRA < 1 it can be shown that introducing unsophisticated consumers into the economy will decrease the rates of interest in both states. In particular, the capital stock in the mixed expectations equilibrium will be larger than in the pure rational expectation equilibrium. Moreover, any factor that increases g^* (e.g. an increase in uncertainty) will increase γ^* and thus increase the rates of interest in the two states. This attempt to endogenize the proportion of fully informed versus imperfectly informed agents in equilibrium, will give some intuition into how changes in this proportion affect the properties of the equilibrium when exogenous changes occur.

6. *Conclusions*

In Haltiwanger and Waldman, 1985, and Possen and Puhakka, 1994, it was found that the mix of as well as the response of sophisticated and unsophisticated consumers had a profound effect on the equilibrium. Here we attempted to enhance our understanding of these models by allowing agents to choose their type. Within the framework of a simple equilibrium model we endogenize the choice of becoming fully informed or remaining unsophisticated. Moreover, we have demonstrated how these proportions change when various exogenous factors in the economy vary. For example, an increase in uncertainty in our economy results in an increase in the proportion of sophisticated agents. These changes in turn affect the equilibrium, the capital stock, and interest rates. In fact, we have shown that results derived in our other paper (1994) with fixed proportions of different types are robust to the endogenization of the decision of whether to become fully rational or not. Introducing these types of interactions in a more macroeconomic framework would seem to be the next important step.

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Appendix

Proof of Proposition 4. By totally differentiating the condition for g^*, $V(c_o, g^*) = V(\hat{c}_o) \equiv \hat{V}$, and taking into account ER = C = constant (as represented by (12)), we obtain

$$\left.\frac{dg^*}{dR_1}\right|_{ER=C} = \frac{\left.\frac{\partial \hat{V}}{\partial R_1}\right|_{ER=C} - \left.\frac{\partial V}{\partial R_1}\right|_{ER=C}}{\frac{\partial V}{\partial g^*}}$$

Differentiating the respective expected utilities and using the envelope theorem we obtain

$$\left.\frac{\partial \hat{V}}{\partial R_1}\right|_{ER=C} = \delta(w_o - \hat{c}_o)\pi[u'(\hat{c}_1^1) - u'(\hat{c}_1^2)] < 0$$

$$\left.\frac{\partial V}{\partial R_1}\right|_{ER=C} = \delta(w_o - c_o - g^*)\pi[u'(c_1^1) - u'(c_1^2)] < 0$$

$$\frac{\partial V}{\partial g^*} = -\delta[\pi R_1 u'(c_1^1) + (1-\pi) R_2 u'(c_1^2)] < 0$$

c_1^i ($\hat{c}_1^i$) [i = 1,2] denote the future consumption of fully rational (unsophisticated consumers in state i. $\left.\frac{dg^*}{dR_1}\right|_{ER = C}$ cannot in general be signed. It will equal the opposite of the sign of the numerator (N). Considering the special case (a) first, we obtain for the numerator

$$N = \delta(w_o - \hat{c}_o)\pi[\mu[R_1(w_o - \hat{c}_o)]^{\mu-1} - \mu[R_2(w_o - \hat{c}_o)]^{\mu-1}] - \delta(w_o - c_o - g^*) \pi[\mu[R_1(w_o - c_o - g^*)]^{\mu-1} - \mu[R_2(w_o - c_o - g)]^{\mu-1}.$$

By simplifying we get

$$N = \delta\pi\mu(w_o - \hat{c}_o)^{\mu}[R_1^{\mu-1} - R_2^{\mu-1}] - \delta\pi\mu(w_o - c_o - g^*)^{\mu}[R_1^{\mu-1} - R_2^{\mu-1}] = \delta\pi\mu[R_1^{\mu-1} - R_2^{\mu-1}][(w_o - \hat{c}_o)^{\mu} - (w_o - c_o - g^*)^{\mu}].$$

Because $R_1 > R_2$ and $w_o - \hat{c}_o > w_o - c_o - g^*$ (as concluded earlier) $N < 0$.

Thus, $\left.\frac{\partial g^*}{\partial R_1}\right|_{ER = C} > 0$.

Case (b). Using the same strategy as above we obtain

$$N = \delta(w_o - \hat{c}_o)\pi[\mu^2[R_1(w_o - \hat{c}_o)]^{-\mu-1} - \mu^2[R_2(w_o - \hat{c}_o)]^{-\mu-1}] - \delta(w_o - c_o - g) \pi[\mu^2[R_1(w_o - c_o - g)]^{-\mu-1} - \mu^2[R_2(w_o - c_o - g)]^{\mu-1}]$$

and simplifying

$$N = \delta\pi\mu^2(w_o - \hat{c}_o)^{-\mu}[R_1^{-\mu-1} - R_2^{-\mu-1}] - \delta\Pi\mu^2(w_o - c_o - g^*)^{-\mu}[R_1^{-\mu-1} - R_2^{-\mu-1}]$$

$$N = \delta\pi\mu^2[R_1^{-\mu-1} - R_2^{-\mu-1}][(w_o - \hat{c}_o)^{-\mu} - (w_o - c_o - g^*)^{-\mu}].$$

In case (b) $w_o - c_o - g^* > w_o - \hat{c}$. Thus $N < 0$. Hence also in case (b)

$$\left.\frac{\partial g^*}{\partial R_1}\right|_{ER = C} > 0.$$

Q.E.D.