DISCRETE EXCHANGE RATE CHANGES WITH REAL WAGE RESISTANCE*

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Fixed exchange rate policy is examined when money wages are determined by collective bargaining for fixed periods. The main results are as follows. If the interest elasticity of aggregate demand is high and the contract periods in the labour market are long, then in the short run, a devaluation produces expansion but in the long run, contraction. Expectations on a future devaluation cause expansion before the occurrence of the devaluation and at the moment of the occurrence, domestic expenditure falls discontinuously.

1. Introduction

This paper focuses on a small economy where wages are centrally determined, foreign and domestic goods are imperfect substitutes, and where the central bank is committed to a fixed exchange rate. We shall consider the labour market conditions reminiscent of those e.g. in Finland, Austria, West Germany, the Netherlands and the Scandinavian countries where the economy wide labour and employer organizations bargain over wages, and where labour contracts are set for fixed periods. Under such conditions, real wage resistance will appear: unions attempt to defend the workers' purchasing power but because of the bargaining process, there are adjustment constraints for wages. This means that after the occurrence of an unanticipated shock, it takes time for the unions to attain their real wage targets. With a centralized wage setting, the

To analyse the problem, some simplifying assumptions turn out to be necessary. The first of these is that the budget constraints of the government and the central bank are integrated, and that taxes can be imposed in a lump sum manner. Then changes in the foreign asset position of the central bank will result in changes in the (lump sum) transfers that the government pays to the citizens, which will completely offset any effects of a change in the foreign asset holdings (and earnings on those) of the private sector on aggregate demand. If the central bank's foreign exchange reserves are insufficient to defend the fixed exchange rate, we would be led to the specula-

increase of import prices leads to wage claims and to anticipated wage-price inflation which in the short run, raises aggregate demand and employment, but in the long run reduces employment through government money illusion. This paper attempts to present a theoretical explanation of this phenomenon.

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¹ Government money illusion is assumed e.g. in Willman (1988).

tive attack hypothesis which is beyond the scope of this paper. Therefore, we assume that because of some unspecified transactions costs, there is an institutionally determined upper limit to capital movements in the case of devaluation expectations,² and that the central bank can always borrow a sufficient amount of reserves from abroad. This eliminates the expectations on a balance-of-payments crisis.³ These simplifications are needed to fully concentrate on the interaction of exchange rate policy and wage-employment dynamics, which is the main focus of this study.

The plan of the paper is as follows. In section 2, the model will be specified by which in 3, the dynamics of the economy will be presented. In 4, the patterns of output and employment will be examined when the exchange rate is depreciated. In particular, the development of wages, employment and domestic expenditure will be analysed.

The basic model

For simplicity, we use a linear model where all parameters are positive and all variables except domestic inflation π and the interest rate r — are logarithmic. This section constructs the instantaneous equilibrium of the economy where output y, the output price p, the nominal wage w and the expected rate of domestic inflation π are given. The supply of imported products is infinitely elastic at price unity, so that the price for importables equals the official exchange rate x. Let $c = \alpha x +$ $(1-\alpha)p$ be the logarithm of the Consumer Price Index and assume that the economy is initially in a steady state. Then without losing any generality, the unit of domestic output can be chosen so that the initial CPI at time t = 0is equal to one: c(t) = 0 for $t \le 0$.

At t = 0, the public receives the information

that a devaluation will be carried out at $t=t_0>0$. Then some time t_1 after the devaluation, the CPI will attain a new steady-state value c^* . Assume that a union attempts to maintain its members' real wage fixed and that its expectations are forward looking. Then applying the Mean Value theorem, the union's target wage, \overline{w} , is obtained as a discounted value of future CPIs:

(1)
$$\overline{w}(t) = v \int_{t}^{\infty} e^{(t-\tau)v} c(\tau) d\tau$$

$$= v \int_{t}^{t_{1}} e^{(t-\tau)v} c(\tau) d\tau + v c^{*} \int_{t_{1}}^{\infty} e^{(t-\tau)v} d\tau$$

$$= v \int_{t}^{t_{1}} e^{(t-\tau)v} c(\tau) d\tau + e^{(t-t_{1})v} c^{*}$$

$$= (t_{1} - t) v e^{(t-\xi)v} c(\xi) + e^{(t-t_{1})v} c^{*},$$

where $0 \le t < \xi \le t_1$. In (1), $v = \frac{\partial \overline{w}(t)}{\partial c(t)}$ is the share of a current price increase that is immediately transferred to the target wage. The smaller is a union's rate of time preference, the lower v is. A union raises its wage at the level of the target \overline{w} as soon as its contract expires. Since the unions' contract periods are overlapping, the nominal wage is formed by the following backward looking scheme:

(2)
$$w(t) = \beta \int_{-\infty}^{t} e^{(\tau - t)\beta} \, \overline{w}(\tau) \, d\tau,$$

where β is the share of workers whose contract expire at t.

The model in (1) and (2) is a combination of the backward looking wage formation by Sachs (1980) and the overlapping contract model by Calvo (1983). If $\overline{w}(t) = c(t)$ is substituted for (1), we obtain the former, and if $w(t) = \overline{w}(t)$ is substituted for (2), we obtain the latter. This model is, however, too complicated to obtain any qualitative results. Since $|t_1-t| \vee \leq t_1 \vee$ in (1), we can approximate $\overline{w} \approx c^*$, if $t_i v \approx 0$. This means that if a union's rate of time preference is low (i.e. v is small) and if the economy attains the new steady state in a relatively short time (i.e. t_1 is relatively small), then the union's target wage is (approximately) the CPI of the new steady state c^* . Since $t_0 \le t_1$, our analysis is valid only for small devaluations that are expected

² The agents need some amount to domestic currency to carry out their daily transactions, and they need collaterals to borrow a large amount of money, for instance. These two assumptions together ensure that an agent cannot transfer an infinite amount of domestic money into foreign currency at a short notice.

³ Kostiainen and Taimio (1988) use the same assumption for the same purpose.

to be carried out in the near future.⁴ Consider first the point of discontinuity at t=0. Given $\overline{w}=c^*$ and c(t)=0 for $t\leq 0$, we obtain $\Delta \overline{w}(0) = \Delta c^* = c^*$, and given (2), we obtain

$$\frac{\partial w(t)}{\partial \overline{w}(0)} = \beta e^{-\beta t}, \ \frac{\partial w(0)}{\partial \overline{w}(0)} = \beta,$$

 $\Delta w(0) = \beta \Delta \overline{w}(0) = \beta c^*$.

Now since (2) is differentiable for t>0, it can be transformed into the form

(3)
$$\dot{w} = \beta [\overline{w} - w] = \beta [c^* - w]$$

for $t > 0$, $\Delta w(0) = \beta \Delta c^* = \beta c^*$,

where

(4)
$$c^* = ax^* + (1 - \alpha)p^*,$$

 $0 < \alpha < 1, \quad 0 < \beta < 1.$

When the public receives the information on a devaluation, the wage level jumps upwards by the amount βc^* . Then wages will increase, until the new equilibrium level c^* is attained.

The price for domestic products p clears the output market. Assume that capital stock is fixed and fully utilized. Then the firms expand output up to the point at which the producer real wage w-p equals marginal product of labour, which falls with output:

$$(5) w-p=-\mu y.$$

Here, μ is the elasticity of the marginal product of labour with respect to output. We assume that the only two assets held by residents are a domestic money not held by foreigners and an internationally traded consol-type bond paying r units of the imported product per unit time in perpetuity. The central bank holds these consols as for-

eign reserves. In the case of devaluation expectations domestic money are exchanged into bonds, but the fall in the central bank reserves will outweigh this exchange, and consequently, the domestic interest rate will always be fixed at the level of r.

Private agents have perfect foresight, but they are still occasionally subject to unanticipated shocks. This means that when a shock occurs, the price for product p (and consequently output y) can make discontinuous jumps but otherwise, actual and expected inflation must coincide: $\dot{p} = \pi$. In line with Willman (1988), we introduce equations

(6)
$$y = \rho[x-p] - \gamma[r-\pi] + g,$$

(7)
$$g = \theta - \sigma p$$
,

where g is the index of fiscal policy, which includes both net transfers and government expenditure. Eq. (6) is a conventional textbook IS curve. We assume that all government transfers and expenditure are financed by taxes, and that the share of expenditure falling on the home good is larger for the government than for the private sector. This means that the index of fiscal policy g tends to raise aggregate demand for home products. Eq. (7) is based on the idea that some items in the government budget are constant in real terms and some items (e.g. interest payments on public debt) are constant in nominal terms. The short run equilibrium conditions are (5), (6) and (7), with inflationary expectations π , the index of fiscal policy g and the domestic price p as endogenous variables. The expected rate of inflation in this system is determined as follows:

(8)
$$\pi(w, y, x, m) = r + \frac{1}{\gamma} \{ [1 + (\sigma + \rho)\mu] y + (\sigma + \rho) w - \rho x - \theta \}.$$

3. Equilibrium dynamics

Two differential equations govern the economy's motion over time: $\pi = \dot{p}$ describes the evolution of expectations regarding domestic inflation and (3) the adjustment of wages in collective bargaining. In the steady state

If two devaluations were carried out successively, then because of $v \approx 0$, it is the CPI after the latter devaluation that matters. The analysis can be easily extended to this case.

⁵ This rough but vastly simplifying assumption is used e.g. in Obstfeld (1982).

⁶ The extension of the analysis to the case of four assets — domestic and foreign money as well as domestic and foreign bonds — is straightforward. The only difference would be that with domestic bonds, there is a scope for independent monetary policy by pegging the domestic interest rate.

 $\dot{p} = \dot{w} = \dot{y} = 0$ holds, where $w = c^*$, $p = p^*$, $y = y^*$ and $x = x^*$. Equations (4) and $\pi = 0$ must hold, and by (5) and (8) they can be changed into the form $\gamma r + [1 + (\sigma + \rho)\mu]y^* + (\sigma + \rho)c^* - \rho x^* - \theta = 0$ and $\alpha(c^* - x^*) + (\alpha - 1)\mu y^* = 0$, where y^* and c^* are endogenous variables. Thus we obtain $y(x^*)$ and $c^*(x^*)$ with

(9)
$$\frac{dy^*}{dx^*} = \frac{\sigma}{1 + (\sigma + \rho)\frac{\mu}{\alpha}} < 0,$$
$$\frac{dc^*}{dx^*} = \frac{1 + (\sigma + \frac{\rho}{\alpha})\mu}{1 + (\sigma + \rho)\frac{\mu}{\alpha}} > 0.$$

Thus in the long run, a devaluation will raise wages but reduce output and employment, which is intuitively clear. With higher import prices, the unions aim at higher wages and domestic prices will rise. This will reduce aggregate demand through government money illusion and lead to a lower level of output and employment.

According to (3), (5), (8) and (9), the time path of output is governed by

(10)
$$\dot{y} = \frac{1}{\mu} (\dot{p} - \dot{w})$$

= $\frac{1}{\mu} \pi(w, y, x, m) + \frac{\beta}{\mu} [w - c^*(x^*)].$

Since this system is wholly independent of p, we can ignore the third differential equation $\dot{p} = \pi$. In the vicinity of the steady state (c^*, y^*) , by (8), equations (3) and (10) become

The characteristic roots of the system (11) are $\lambda_1 = -\beta < 0$ and $\lambda_2 = \frac{1}{\mu} \pi_y > 0$. In the vicinity of (c^*, y^*) , relations (3) and (8) – (11) imply

(12)
$$\frac{\partial \dot{w}}{\partial w} < 0, \quad \frac{\partial \dot{w}}{\partial y} = \frac{\partial \dot{w}}{\partial x} = 0, \quad \frac{\partial \dot{w}}{\partial x^*} > 0,$$
$$\frac{\partial \dot{y}}{\partial w} = \phi > 0, \quad \frac{\partial \dot{y}}{\partial y} = \eta > 0, \quad \frac{\partial \dot{y}}{\partial x} < 0,$$
$$\frac{\partial \dot{y}}{\partial x^*} < 0.$$

This shows that on (w, y)-plane, curve $(\dot{w} = 0)$ is vertical and curve $(\dot{y} = 0)$ downward sloping. In Figure 1, the saddle path is denoted by TT. Below TT, there is excess demand for and above TT, excess supply of domestic products. Since the producers have perfect foresight, at the given level of wages w, prices and output y adjust so that the market of domestic products is always cleared and the economy is on the path TT. After an exogenous change in the system (3) and (10), output y jumps from the initial level at the level of the path TT and the system proceeds along TT to the steady state (c^*, y^*) .

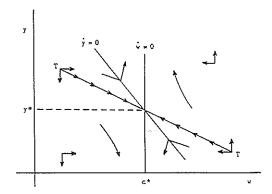


Figure 1. The saddle property of the model.

4. The effects of a devaluation

This section considers a devaluation which raises the exchange rate from x to x^* at $t=t_0\geq 0$, and which is anticipated by the public from t=0. A surprise devaluation is obtained as a special case when $t_0=0$. In time interval $0\leq t < t_0$, devaluation expectations give rise to capital movements, but because of some unspecified transactions costs, there is

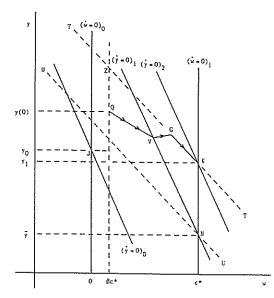


Figure 2. The adjustment when a future devaluation is expected.

an upper limit to these movements. If the central bank can obtain a sufficient amount of reserves, e.g. by borrowing from abroad, it can maintain the fixed exchange rate target.

The system under examination is (3) and (10). The economy is initially at rest so that the equilibrium (logarithmic) CPI and the equilibrium (logarithmic) wage are equal to zero. in Figure 2, we denote the initial steady state by $(0, y_0)$ or J. Now by (12), the development of the system will be as follows. At t=0, the anticipated level of the future exchange rate x* jumps upwards, shifting curve $(\dot{w}=0)$ to the right from $(\dot{w}=0)_0$ to $(\dot{w}=0)_1$, and curve $(\dot{y} = 0)$ upwards from $(\dot{y} = 0)_0$ to $(\dot{y}=0)_1$. At $t=t_0>0$ when the devaluation is actually carried out, curve ($\dot{w} = 0$) remains in position $(\dot{w} = 0)_1$ but curve $(\dot{y} = 0)$ shifts further upwards from $(\dot{y}=0)_1$ to $(\dot{y}=0)_2$. By (9), the increase in x reduces the steady state level of output: $y_0 > y_1$. Point N — which corresponds to the steady state where the target wage is set by the new exchange rate, but where the exchange rate has not yet been changed — governs the system during period $0 < t < t_0$. The saddle path to N is UU, and that to the new steady state K is TT. In period $0 < t < t_0$, the trajectories of the system turn up to the North-East when above UU,

and down to the South-East when below $UU.^7$ At t=0, by (3), the level of wages change from 0 to βc^* . The problem is to find for period $0 < t < t_0$ such a trajectory which reaches the path TT exactly at $t=t_0$. In Figure 2, this trajectory starts at point $Q.^8$ At t=0 when new information is received, output jumps from y_0 to y(0). Then the system proceeds along QVG corresponding to the steady state N, until exactly at $t=t_0$, the path TT is attained at point G. After $t=t_0$, the system proceeds along TT to T.

Since output can respond immediately and discontinuously to exogenous changes, in the short run, it can change in the opposite direction than in the long run. The sufficient condition for the short run expansion is that in *Figure 2*, the saddle path UU leading to N goes above $(\beta c^*, y_0)$. Now, we show that this condition is equivalent to

(13)
$$\beta + \frac{\sigma + \rho}{\gamma} < \frac{1}{1 + (\sigma + \rho)\mu}$$
.

In period $0 < t < t_0$, when $x < x^*$, the system is heading towards a steady state (c^*, \bar{y}) , where by (8), the condition $\gamma \pi = [1 + (\sigma + \rho)\mu]\bar{y} + (\sigma + \rho)c^* - \rho x + \gamma r - \theta = 0$ must hold. Using this and (9), we obtain

(14)
$$\frac{d\overline{y}}{dx^*} = -\frac{\sigma + \rho}{1 + (\sigma + \rho)\mu} \frac{dc^*}{dx^*}.$$

Let $t = \tau$ be any initial moment. Then the solution of the system (11) is⁹

⁷ This can be seen from Figure 1 when UU is substituted for TT, and N for (c*, y*).

⁸ Here both the wage level w and output y change discontinuously at t=0. This is, however, consistent with the use of a model containing one stable and one unstable root. The public know that a fixed share β of labour contracts expire at t=0, when a new information on a devaluation is received, and therefore, they expect that the wage level jumps from 0 to β c*. Then at the given wage β c*, domestic firms meet the demand by increasing output from y_0 to y(0), so that the new steady state K can be attained. In other words, the change of the wage from 0 to β c* at t=0 is exogenous, while the roots of the system (3) and (10) must be such that on the condition output y and the price y can jump at y constable and one unstable root.

⁹ Hirsch and Smale (1974).

(15)
$$w(t) = c^* + e^{-\beta(t-\tau)}[w(\tau) - c^*],$$

(16)
$$y(t) = y^* + \left\{ \frac{\phi}{\beta + \eta} [w(\tau) - c^*] + y(\tau) - y^* \right\} e^{\eta(t - \tau)} - \frac{\phi}{\beta + \eta} [w(\tau) - c^*] e^{-\beta(t - \tau)}.$$

Since $\beta > 0$ and since by (12), $\eta > 0$, we see that when $t \to \infty$, then $w \to c^*$ always but $y \to y^*$ only if $\frac{\phi}{\beta + \eta}[w(\tau) - c^*] + y(\tau) - y^* = 0$. By this and by (11), and by substituting t for τ and (c^*, \bar{y}) for (c^*, y^*) , the saddle path UU leading to (c^*, \bar{y}) can be constructed:

(17)
$$y(t) = \bar{y} - \frac{\phi}{\beta + \eta} [w(t) - c^*]$$

= $\bar{y} - \frac{\pi_w + \beta}{\pi_y + \beta \mu} [w(t) - c^*].$

By (8) and (13)-(17), we obtain

(18)
$$\frac{\pi_{w} + \beta}{\pi_{y} + \beta \mu} = \frac{\sigma + \rho + \gamma \beta}{1 + (\sigma + \rho + \gamma \beta) \mu}$$
$$> \frac{1}{1 - \beta} \frac{\sigma + \rho}{1 + (\sigma + \rho) \mu}$$
$$= -\frac{\frac{d\overline{y}}{dx^{*}}}{(1 - \beta) \frac{dc^{*}}{dx^{*}}}.$$

The line UU in Figure 2 is given by (17). The left hand side of (18) is the absolute value of the slope for UU and the right hand side is that for the line connecting point $(\beta c^*, y_0)$ and N. Thus UU must fall more steeper and therefore go over point $(\beta c^*, y_0)$. So we have proven (13) to be the sufficient condition for the short-run expansion.

If the interest elasticity γ of aggregate demand is high enough, and if the share β of contracts expiring at t=0 is small enough, then (13) holds and a devaluation, expected or not, has in the short run an expansive impact on output. This can be explained as fol-

lows. Since the union's target wage c^* rises immediately at t=0, expectations on wage price-inflation increase aggregate expenditure and the demand for domestic output. The longer are the overlapping contract periods in the labour market, the smaller is the share B of labour contracts expiring at t = 0, the greater is the remaining wage increase $(1-\beta)c^*$ to come and the stronger are inflationary expectations. On the other hand, the output price jumps upwards to clear the market. If the interest elasticity y of aggregate demand is high enough, the effect of inflationary expectations outweighs that of the price level, so that in the short run output will increase. However in the long run, when wages attempt to reach higher consumer prices, output and employment fall gradually. After the occurrence of the devaluation, wages will keep on rising and output will keep on falling, until the new steady state K is attained. So although the long run effect of a devaluation is contractionary, a fairly general condition (13) guarantees that the short run effect is expansionary and part of this expansion occurs before the change of the exchange rate.

Since in Figure 2, w gradually rises from βc^* to c^* , it can be used as a measure of time. In Figure 2, the adjustment path for the case of a surprise devaluation, $t_0 = 0$, would be QZK. Thus the area QZGVQ shows the loss for the postponement of the devaluation. If the length of period $(0, t_0)$ increases, point G shifts farther from Z on the path TT and QZGVQ becomes larger. So the loss increases with the time from the change of expectations to the occurrence of the devaluation. This is because the unions' target wages rise when price increases are expected, causing a constant upward pressure on production costs. As long as the devaluation is anticipated but not vet carried out, the terms of trade will remain at the initial level and the increase in costs will reduce output.

If the price level were expected to jump upwards, the agents would protect themselves against this by exchanging their nominal assets into commodity stores, which would destabilize the goods market. Due to this source of instability, and due to the fact that the agents expect the goods market to clear, the price level and consequently output y cannot jump discontinuously, if t > 0. This means that at $t = t_0$ when the exchange rate x is

raised, output y cannot jump to meet the increased export demand and therefore, domestic expenditure must fall discontinuously. At time $t = t_0$, the increase in the exchange rate x reduces the expected rate of inflation π (see eq. (8)) sharply, so that the real interest rate $r-\pi$ increases and domestic consumption decreases discontinuously. So far, we have assumed that only consumer goods are imported. With imported inputs, the dynamics of the model would otherwise be the same as before, except that the short run expansion due to a devaluation would weaker. This is because the production costs would increase. thus partly offsetting the producer's benefit from higher export prices.

5. Concluding remarks

This paper investigated the effects of a devaluation in a dynamic context where the agents' expectations are rational, and with the following institutional assumptions. First, domestic and foreign goods are imperfect substitutes, so that the domestic price is endogenous and purchasing power parity need not hold. Secondly, some items in the government budget (e.g. interest payments on the public debt) are fixed in nominal terms, so that even in the long run changes in domestic prices have real effects. Thirdly, the central bank can obtain enough reserves, e.g. by borrowing from abroad, to maintain the fixed exchange rate target. Fourthly, the unions attempt to maintain their real wage targets in the long run, but in the short run when contracts are binding, wage adjustment is incomplete. This resembles the institutions of countries with a centralized wage setting, and it enables different outcomes from a devaluation in the short and long runs. We summarize the main results of this paper as follows.

The dynamic analysis of this paper suggests that when aggregate demand is interest elastic and the contract periods in the labour market are long, there is a trade-off between the short and long run effects of a devaluation. Then with an increase in import prices, two things happen. First, the terms of trade fall and the demand for home goods increases. Secondly, since everybody knows that wages will gradually adjust to higher consumer

prices, wage-price inflation is generally anticipated, the expected real interest rate falls and aggregate demand increases. To clear the market for home products, the domestic price must rise and because of sticky money wages, output and employment rise. In the long run, a devaluation will raise prices. This will lead to lower expenditure, a lower demand for domestic goods and consequently, through government money illusion, to a lower level of output and employment. So in the short run, a devaluation produces expansion but in the long run, contraction.

Expectations on a future devaluation raise output and employment already before the occurrence of the devaluation, and at the moment of the occurrence domestic expenditure falls discontinuously. The intuitive explanation behind this is as follows. At the moment new information on a future devaluation is received, unions start pressing higher wages in response to the anticipated price increases and consequently, wage-price inflation is generally expected already before the moment the exchange rate is actually changed. These inflationary expectations raise expenditure and aggregate demand, so that domestic output and employment must increase. The short run expansion is, however, smaller than in the case of a surprise devaluation for the following reason: with expectations on price increases and with the original exchange rate, wages tend to rise causing an increasing strain on production costs. Therefore, the later the expected devaluation is postponed, the larger amount of output will be lost. When the devaluation is carried out, inflationary expectations will fall increasing the real interest rate and decreasing sharply domestic demand.

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Appendix

Inequalities (13) and (18) are equivalent as follows:

$$\frac{\sigma + \rho + \gamma \beta}{1 + (\sigma + \rho + \gamma \beta)\mu} > \frac{1}{1 - \beta} \frac{\sigma + \rho}{1 + (\sigma + \rho)\mu} \iff \frac{1}{\sigma + \rho + \gamma \beta} + \beta \mu < \frac{1 - \beta}{\sigma + \rho}$$

$$\iff \beta \mu < \frac{1 - \beta}{\sigma + \rho} - \frac{1}{\sigma + \rho + \gamma \beta} = \frac{(1 - \beta)(\sigma + \rho + \gamma \beta) - \sigma - \rho}{(\sigma + \rho)(\sigma + \rho + \gamma \beta)} = \beta \frac{(1 - \beta)\gamma - \sigma - \rho}{(\sigma + \rho)(\sigma + \rho + \gamma \beta)}$$

$$\iff \mu < \frac{(1 - \beta)\gamma - \sigma - \rho}{(\sigma + \rho)(\sigma + \rho + \gamma \beta)} \iff (\sigma + \rho)(\sigma + \rho + \gamma \beta)\mu < (1 - \beta)\gamma - \sigma - \rho$$

$$\iff [(\sigma + \rho)\mu + 1]\beta\gamma < \gamma - \sigma - \rho - (\sigma + \rho)^2 \mu \iff \beta + \frac{\sigma + \rho}{\gamma} < \frac{1}{1 + (\sigma + \rho)\mu}.$$

We prove formally that in Figure 2, point Q is above the line UU. Then condition (13) implies that the line UU is above point $(\beta c^*, y_0)$, and that output will expand immediately after the public receives information on a future devaluation.

For interval $0 \le t \le t_0$, the initial value τ equals zero and the steady state governing the dynamics is given by (c^*, \vec{y}) , and for interval $t \ge t_0$, τ equals t_0 and the governing steady state is given by (c^*, y_1) . Then noting $w(0) = \beta c^*$, (15) and (16), we obtain

$$\begin{split} w(t_0) &= c^* + e^{-\beta t_0} [w(0) - c^*] = [1 + (\beta - 1)e^{-\beta t_0}]c^*, \\ y(t) &= y_1 - \frac{\phi}{\beta + \eta} [w(t_0) - c^*] e^{-\beta(t - t_0)} \quad \text{for } t \ge t_0, \\ y(t) &= \bar{y} + \{ \frac{\phi}{\beta + \eta} [w(0) - c^*] + y(0) - \bar{y} \} e^{\eta t} - \frac{\phi}{\beta + \eta} [w(0) - c^*] e^{-\beta t} \\ &= \bar{y} + \{ \frac{(\beta - 1)\phi}{\beta + \eta} c^* + y(0) - \bar{y} \} e^{\eta t} - \frac{(\beta - 1)\phi}{\beta + \eta} c^* e^{-\beta t} \quad \text{for } 0 \le t \le t_0. \end{split}$$

The continuity of y at $t = t_0$ implies

$$y_{1} - \frac{(\beta - 1)\phi}{\beta + \eta} e^{-\beta t_{0}} c^{*} = y_{1} - \frac{\phi}{\beta + \eta} [w(t_{0}) - c^{*}] = \lim_{t \to t_{0} +} y(t) = \lim_{t \to t_{0} +} y(t)$$

$$= \bar{y} + \{ \frac{(\beta - 1)\phi}{\beta + \eta} c^{*} + y(0) - \bar{y} \} e^{\eta t_{0}} - \frac{(\beta - 1)\phi}{\beta + \eta} c^{*} e^{-\beta t_{0}}$$

$$= (1 - e^{\eta t_{0}}) \bar{y} + \frac{(\beta - 1)\phi}{\beta + \eta} (e^{\eta t_{0}} - e^{-\beta t_{0}}) c^{*} + y(0) e^{\eta t_{0}}$$

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and

(19)
$$y(0) = e^{-\eta t_0} y_1 + (1 - e^{-\eta t_0}) \bar{y} + \frac{(1 - \beta) \phi}{\beta + \eta} c^*.$$

Given (12), we know that

$$y_1 - \bar{y} = \frac{dy}{dx}\Big|_{\dot{y}=0} (x^* - x) > 0.$$

Denoting the value of the line UU or (17) at $w = \beta c^*$ by $y^u = \overline{y} + \frac{(1-\beta)\phi}{\beta + \eta} c^*$, and noting (19), we obtain that point Q is above UU:

$$y(0) = y^{u} + e^{-\eta t_0}(y_1 - \bar{y}) > y^{u}.$$